REVIEW II

1.

Problem 1. Compute the ranks of the following matrices:

(a)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)

$$B = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(c)

$$C = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

(d)

$$D = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \\ 3 & 0 & 3 \end{bmatrix}$$

2.

Problem 2. Suppose V is the vector space of all sequences that repeat every second term, and consider the ordered basis $\mathcal{B} = [b^{(1)}, b^{(2)}]$ with

$$b^{(1)} = (1, 2, 1, 2, 1, 2, 1, 2, 1, 2 \ldots)$$

and

$$b^{(2)} = (2, 2, 2, 2, 2, 2, 2, 2, 2, 2 \ldots).$$

What is the coordinate vector, with respect to $\mathcal{B}$, of each of the following:

(a)

$$c = (2, 4, 2, 4, 2, 4, \ldots)$$

(b)

$$d = (0, 0, 0, 0, 0, 0, \ldots)$$

(c)

$$c = (0, 1, 0, 1, 0, 1, \ldots)$$

3.

Problem 3.

(a) Find all solutions to

$$r\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} + s\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} + t\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

(b) Find all solutions to

$$r\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} + s\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} + t\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

(c) Is

$$\left\{\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right\}$$

the basis of some subspace of $\mathbb{R}^{2,2}$?

4.

Problem 4. Suppose $T : V \to \mathbb{R}^3$ is linear and that

$$T(\mathbf{v}_1) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},$$

$$T(\mathbf{v}_2) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

and

$$T(\mathbf{v}_3) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(a) What is $T(\mathbf{v}_1 + \mathbf{vs}_3)$?
(b) What is $T(4\mathbf{v}_3)$?
(c) Assuming that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent, find a vector $\mathbf{x}$ other than 0 so that $T(\mathbf{x}) = 0$.

5.

Problem 5. Suppose $T : V \to W$ is a linear transformation and that $\mathcal{B} = [\mathbf{b}_1, \mathbf{b}_2]$ is an ordered basis for V and that $\mathcal{C} = [\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3]$ is an ordered basis for V. Suppose the matrix representing T relative to $\mathcal{B} - \mathcal{C}$ is

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 2 & 0 \end{bmatrix}.$$

(a) What is the dimension of V?
(b) What is $T(\mathbf{b}_1)$?
(c) If $\mathbf{v}$ has $\mathcal{B}$-coordinates

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix},$$

what are the $\mathcal{C}$-coordinates of $T(\mathbf{v})$?

6.

Problem 6. For each matrix called A, find a basis for $N(A)$ and $\mathrm{col}(A)$.

(a)

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}.$$