Problem 0.1. Find all solutions in $x_1, \ldots, x_4$ to the simultaneous sets of equations

$$
\begin{aligned}
x_1 &&&&&&&& = y_1 \\
2x_1 &+& x_2 &&&&&& = y_2 \\
x_1 &+& 3x_2 &+& x_3 &&&& = y_3 \\
x_1 &+& x_2 &+& x_3 &+& x_4 & = & y_4
\end{aligned}
$$

and

$$
\begin{aligned}
y_1 &+& y_2 &+& y_3 &+& y_4 &=& 3 \\
&& y_2 &+& 4y_3 &+& 2y_4 &=& 12 \\
&&&& y_3 &+& y_4 &=& 4 \\
&&&&&& y_4 &=& 3
\end{aligned}
$$

Problem 0.2. Verify that the inverse of

$$
\begin{bmatrix}
2 & 2 & 0 & 1 \\
2 & 1 & 0 & 1 \\
-1 & 2 & -1 & 0 \\
1 & 1 & 0 & 0
\end{bmatrix}
$$

is one of

$$
\begin{bmatrix}
-1 & 1 & 0 & 1 \\
1 & -1 & 0 & 0 \\
1 & 0 & 0 & -2 \\
3 & -3 & -1 & -1
\end{bmatrix}
\text{ or }
\begin{bmatrix}
-1 & 1 & 0 & 1 \\
1 & -1 & 0 & 0 \\
3 & -3 & -1 & -1 \\
1 & 0 & 0 & -2
\end{bmatrix}
\text{ or }
\begin{bmatrix}
1 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 \\
3 & -3 & -1 & -1 \\
1 & 0 & 0 & -2
\end{bmatrix}.
$$

Knowing this, solve

$$
\begin{aligned}
2x_1 &+& 2x_2 &&&+& x_4 &=& 1 \\
2x_1 &+& x_2 &&&+& x_4 &=& 0 \\
-x_1 &+& 2x_2 &-& x_3 &&&=& 1 \\
x_1 &+& x_2 &&&&&=& 1
\end{aligned}
$$

Problem 0.3. Find all solutions to

$$
\begin{array}{rrrrrrrrrl}
x_1 & + & x_2 & + & 3x_3 & + & 6x_4 & + & 3x_5 & = & -4 \\
-x_1 & - & x_2 & + & 2x_3 & + & 4x_4 & - & 2x_5 & = & -6 \\
& & & & -x_3 & - & 2x_4 & - & x_5 & = & 3
\end{array}
$$

Problem 0.4. Find all solutions to

$$
\begin{array}{rrrrrrrrl}
x_1 & + & x_2 & + & 3x_3 & + & 2x_4 & + & 2x_5 & = & 5 \\
-x_1 & - & x_2 & + & 2x_3 & + & 3x_4 & + & 3x_5 & = & 5 \\
& & & & -x_3 & - & 2x_4 & - & x_5 & = & -2
\end{array}
$$

Problem 0.5. Find the inverse to each of the following matrices:

(a)
$$
A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}
$$

(b)
$$
B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

(c)
$$
C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

(d)
$$
D = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

(e)
$$
F = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Problem 0.6. Find the determinent of A where
$$A = \begin{bmatrix} -3 & 2 & 3 \\ 0 & 1 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} -3 & 2 & 3 \\ 0 & 1 & 2 \\ 4 & 0 & 1 \end{bmatrix}^{-1}$$

Problem 0.7. Find the determinant of the following matrix, in terms of the unknow reals r and s.
$$A = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 2r & 1 & r & r \\ 1 & 0 & s & 1 \\ 4 & 1 & 1 & 3 \end{bmatrix}$$

Problem 0.8. Find elementary matrices whose product equals A, where
$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$