

HOMEWORK 9

1.

Problem. §5.5, Problem #5

2.

Problem. §5.5, Problem #33 (You can replace “inner product space” by “ \mathbb{R}^n ” and “ $\langle \mathbf{a}, \mathbf{b} \rangle$ ” by “ $\mathbf{a} \cdot \mathbf{b}$.”)

3.

Problem. Suppose A is an n -by- n matrix of rank n . Suppose there is a matrix B (also n -by- n) so that

$$B^T = B$$

and

$$B^2 = AA^T.$$

(a) Show that B is invertible.

(b) Show that

$$O = B^{-1}A$$

is orthogonal.

For the first part, you will need to use this result, stated in Problem 13, section 5.2:

Fact: For any matrix X ,

$$\text{rank}(X) = \text{rank}(X^T X)$$

4.

Problem. §6.1, Problem #8

5.

Problem. Suppose A is a three by three matrix, and that

$$A \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix},$$

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

and

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}.$$

(a) Find matrices D and X with D diagonal and so that

$$A = XDX^{-1}.$$

(b) Find

$$\det(A).$$

6.

Problem. §6.1, Problem #4

7.

Problem. Find all the eigenvalues, and a eigenvector for each eigenvector, for

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 1 \end{bmatrix}.$$