HOMEWORK

Problem 1. Let

$$A = \begin{bmatrix} 1 & -2 & -3 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 1 & -2 & -3 & 0 & 0 & 2 \end{bmatrix}.$$

(a) Find a basis for the null space of A.
(b) Find a basis for the column space of A.
(c) What is the rank of A?
(d) What is the nullity of A^T?

Problem 2. Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & -4 & 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & -1 & 0 & 2 \end{bmatrix}.$$

(a) Find a basis for the row space of A.
(b) Find a basis for the column space of A.

Problem 3. Consider the set of vectors comprised of

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 4 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{v}_4 = \begin{bmatrix} 1 \\ 5 \\ -1 \\ 6 \end{bmatrix}, \ \mathbf{v}_5 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 6 \end{bmatrix}.$$

Find a subset of this set of vectors that is a basis for their span.

Problem 4. Number 8 in §4.1.

Problem 5. Consider V with ordered basis

$$\mathcal{B} = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$$

and W with ordered basis

$$\mathcal{C} = [\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3]$$

Suppose T is a linear transformation from V to W. Suppose

$$T(\mathbf{b}_1) = \mathbf{c}_1 + 2\mathbf{c}_2$$

and

$$T(\mathbf{b_1} + \mathbf{b}_2) = \mathbf{c}_1 + 3\mathbf{c}_3$$

and

$$T(\mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3) = 3\mathbf{c}_1 + 4\mathbf{c}_2.$$

(a) What is the matrix representing T with respect the the bases $\mathcal{B}$ and $\mathcal{C}$?

(b) What is the matrix representing T with respect the the bases

$$[\mathbf{b}_1, \mathbf{b}_1 + \mathbf{b}_2, \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3]$$

and $\mathcal{C}$?

(Correction: Added c_3 to $\mathcal{C}$.)

Problem 6. Consider the linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^4$ defined by

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \\ a \end{bmatrix}.$$

(a) What matrix A satisfies

$$T(\mathbf{v}) = A\mathbf{v}$$

for all $\mathbf{v}$ in $\mathbb{R}^3$?

(b) With respect to the ordered basis

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{b}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \ \mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

and the ordered basis

$$\mathbf{c}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{c}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{c}_4 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix},$$

what is the matrix representation of T?

(Corrections: dropped extra parentheses in equation defining T. Change the second c_3 into a c_4.)