HOMEWORK 6

Problem 1. Find the solution for the following two sets of linear equations. Do so by row reducing a single matrix to row echelon form and then back-soving separately.

$$\begin{array}{rcrcrcr} -x_1 & + & 2x_2 & + & x_3 & = & -1 \\ 1x_1 & - & 3x_2 & & & = & 0 \\ -x_1 & + & 2x_2 & - & x_3 & = & -3 \end{array} \qquad \Bigg| \qquad \begin{array}{rcrcrcr} -x_1 & + & 2x_2 & + & x_3 & = & 0 \\ 1x_1 & - & 3x_2 & & & = & 0 \\ -x_1 & + & 2x_2 & - & x_3 & = & -2 \end{array}$$

(I've posted an example similar to this.)

Problem 2. Let

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 1 & 0 & 0 \\ 9 & -3 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 4 & -2 \end{bmatrix}.$$

(a) Find all solutions to

$$AX = B$$

by reducing the matrix

$$\begin{bmatrix} A & B \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 9 & -3 & 1 & 4 & -2 \end{bmatrix}$$

to reduced echelon form and extracting the right-most two columns.

(b) Find all solutions to

$$AX = B$$

by computing A^{-1} in the usual way and multiply this against B.

(c) If B had been some 3-by-1000 matrix, which method would be less work?

For the last part, you don't need to compute the exact number of multiplications and additions to be done. Just estimate how many numbers you would need to write down for each method.

Problem 3. Consider the vector space V of strictly upper triangular 3-by3 matrices

$$V = \left\{ \begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix} \middle| \, a, b, c \text{ are real numbers} \right\}.$$

Consider also the ordered basis $\mathcal{B}$ consisting of

$$B_1 = \begin{bmatrix} 0 & 6 & 6 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 & 6 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 & 6 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and finally the ordered basis $\mathcal{C}$ consisting of

$$C_1 = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a linear combination of C_1, C_2 and C_3 that equals B_1.
(b) Find a linear combination of C_1, C_2 and C_3 that equals B_2.
(c) Find a linear combination of C_1, C_2 and C_3 that equals B_3.
(d) What is the dimension of V.
(e) Find the transition matrix corresponding to the change of basis from $\mathcal{B}$-coorinates to $\mathcal{C}$-coordinates.
(f) If X has $\mathcal{B}$-coordinates

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},$$

what are the $\mathcal{C}$-coordinates of X?

Problem 4.

(a) Consider the vector space V of infinite sequences that are periodic of order 2 :

$$V = \left\{ (a, b, c, a, b, c, \dots) \middle| a, b, c \text{ are real numbers} \right\}.$$

Consider also the ordered basis $\mathcal{B}$ consisting of

$$\begin{aligned} b^{[1]} &= (0, 1, 0, 1, \dots), \\ b^{[2]} &= (1, 1, 1, 1, , \dots), \end{aligned}$$

and finally the ordered basis $\mathcal{C}$ consisting of

$$\begin{aligned} c^{[1]} &= (1, 2, 1, 2, \dots), \\ c^{[2]} &= (1, -1, 1, -1, \dots), \end{aligned}$$

(b) Find a linear combination of c_1 and c_2 that equals b_1.
(c) Find a linear combination of c_1 and c_2 that equals b_1.
(d) Find the transition matrix corresponding to the change of basis from $\mathcal{B}$ coorinates to $\mathcal{C}$ coordinates.

Problem 5.

(a) Find all solutions to the equation
$$r\begin{bmatrix}1\\0\\0\\1\\2\end{bmatrix}+s\begin{bmatrix}1\\2\\1\\2\\0\end{bmatrix}+t\begin{bmatrix}1\\4\\2\\3\\-2\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}.$$

(b) Is the set
$$\left\{\begin{bmatrix}1\\0\\0\\1\\2\end{bmatrix},\begin{bmatrix}1\\2\\1\\2\\0\end{bmatrix},\begin{bmatrix}1\\4\\2\\3\\-2\end{bmatrix}\right\}$$
linearly idependant?

(c) Find all solutions to the equation
$$r\begin{bmatrix}1\\0\\0\\1\\2\end{bmatrix}+s\begin{bmatrix}1\\2\\1\\2\\0\end{bmatrix}+t\begin{bmatrix}1\\4\\2\\3\\-2\end{bmatrix}=\begin{bmatrix}1\\2\\2\\2\\1\end{bmatrix}.$$

(d) Is
$$\begin{bmatrix}1\\2\\2\\2\\1\end{bmatrix}$$
in the span of the set
$$\left\{\begin{bmatrix}1\\0\\0\\1\\2\end{bmatrix},\begin{bmatrix}1\\2\\1\\2\\0\end{bmatrix},\begin{bmatrix}1\\4\\2\\3\\-2\end{bmatrix}\right\}?$$