HOMEWORK 5

Problem 1. In this problem, we will work always in V, where V is the x-y-plane in $\mathbb{R}^3$, which is

$$V = \left\{ \left[\begin{array}{c} x \\ y \\ 0 \end{array} \right] \middle| \, x \text{ and } y \text{ are any real numbers} \right\}.$$

We will use different bases of V to create different coordinate systems.

(a) What is the coordinate vector for

$$\left[\begin{array}{c} 2 \\ 3 \\ 0 \end{array} \right]$$

with respect to the ordered basis

$$\left[\left[\begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right] \right]?$$

(b) What is the coordinate vector for

$$\left[\begin{array}{c} 2 \\ 3 \\ 0 \end{array} \right]$$

with respect to the ordered basis

$$\left[\left[\begin{array}{c} 10 \\ 10 \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ 10 \\ 0 \end{array} \right] \right]?$$

(c) What is the coordinate vector for

$$\left[\begin{array}{c} 2 \\ 3 \\ 0 \end{array} \right]$$

with respect to the ordered basis

$$\left[\left[\begin{array}{c} 2 \\ 2 \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right] \right]?$$

(d) What is $\mathbf{v}$ if $\mathbf{v}$ has coordinate vector

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

with respect to the ordered basis

$$\left[\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right] ?$$

(e) What is $\mathbf{v}$ if $\mathbf{v}$ has coordinate vector

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

with respect to the ordered basis

$$\left[\begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} \right] ?$$

Problem 2. Let

$$\mathbf{b}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Now consider the ordered basis $[\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$ of the subspace of 5-vectors that are symmetric with repect to a vertical flip.

With respect to this ordered basis:

(a) What is the coordinate vector of

$$\begin{bmatrix} 5 \\ 4 \\ 3 \\ 4 \\ 5 \end{bmatrix}?$$

(b) What is the coordinate vector of

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}?$$

(c) Which 5-vector has coordinate vector

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}?$$

(d) Which 5-vector has coordinate vector

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}?$$

Problem 3. Consider the ordered basis $[b_1, b_2, b_3, b_4]$ of $\mathbb{R}^4$:

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{b}_4 = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}.$$

With respect to this basis:

(a) What is the coordinate vector for

$$\begin{bmatrix} 4 \\ 6 \\ 4 \\ 2 \end{bmatrix}?$$

(b) What is the coordinate vector for

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}?$$

Problem 4. Suppose $\mathbf{v}$, and $\mathbf{w}$ are elements of the same vector space V. Find infinitely many solutions in the real variables r, s and t, to the equation

$$r(\mathbf{v}+\mathbf{w})+s(\mathbf{v}-\mathbf{w})+t(2\mathbf{v}-\mathbf{w})=2\mathbf{v}+5\mathbf{w}.$$

Problem 5. Consider the following vector space, which is a subspace of $\mathbb{R}^6$:

$$V=\left\{\left.\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix}\right| x_1=-x_3=x_4=x_5\right\}$$

An ordered basis for this is $[\mathbf{b}_1,\mathbf{b}_2]$ where

$$\mathbf{b}_1=\begin{bmatrix}1\\0\\-1\\1\\1\\1\end{bmatrix},\quad \mathbf{b}_2=\begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}.$$

With respect to this basis:

(a) What is the coordinate vector for

$$5(\mathbf{b}_1+3\mathbf{b}_2)?$$

(b) What is the coordinate vector for

$$\mathbf{b}_2+2\mathbf{b}_1?$$

(c) What is the coordinate vector for

$$\mathbf{b}_1+2\mathbf{b}_1?$$

Problem 6. Let V be the functions on the real line that solve the differential equation

$$f'' = -f.$$

An ordered basis for V is $[f_1, f_2]$ where

$$f_1(x) = \cos(x), \quad f_2(x) = \sin(x).$$

For example, this makes

$$5\sin(x)$$

the solution with coordinate vector

$$\begin{bmatrix} 0 \\ 5 \end{bmatrix}.$$

Since

$$\frac{d}{dx}(\sin(x+1)) = \cos(x+1)$$

and

$$\frac{d}{dx}(\cos(x+1)) = -\sin(x+1),$$

the function given by

$$\sin(x+1)$$

is in V. With respect the basis above, what is its coordinate vector?