HOMEWORK 4

Problem 0.1.

(a) Find one instance of real numbers r and s so that
$$r\begin{bmatrix}1\\1\end{bmatrix}+s\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}6\\8\end{bmatrix}.$$

(b) Find one instance of real numbers r and s so that
$$r\begin{bmatrix}1\\1\\1\end{bmatrix}+s\begin{bmatrix}1\\1\\-1\end{bmatrix}=\begin{bmatrix}6\\6\\8\end{bmatrix}.$$

(c) Find one instance of real numbers r and s and t so that
$$r\begin{bmatrix}1\\1\\1\end{bmatrix}+s\begin{bmatrix}1\\1\\-1\end{bmatrix}+t\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}6\\6\\8\end{bmatrix}.$$

(d) Find *all possible triples* of real numbers r and s and t so that
$$r\begin{bmatrix}1\\1\\1\end{bmatrix}+s\begin{bmatrix}1\\1\\-1\end{bmatrix}+t\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}6\\6\\10\end{bmatrix}.$$

(e) Show that there are no triples of real numbers r, s and t so that
$$r\begin{bmatrix}1\\1\\1\end{bmatrix}+s\begin{bmatrix}1\\1\\-1\end{bmatrix}+t\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}6\\1\\8\end{bmatrix}.$$

Problem 0.2.

(a) Find all solutions in r and s to the following equation between polynomials:
$$r(x^2+x-1)+s(x^2-x-1)=4x^2+2x-4.$$

(b) Find all solutions in r and s to the following equation between polynomials:
$$r(x^2+x-1)+s(x^2-x-1)=4x^2+2x+1.$$

Problem 0.3. Number 11 on page 132.

Problem 0.4. Given the following elements of P^5,

$$\begin{aligned} p_1(x) &= -1x^4 + 2x^3 + 3x^2 \\ p_2(x) &= 3x^4 + 4x^3 + 2x^2 \\ q(x) &= 2x^4 + 6x^3 + 6x^2 \\ f(x) &= -9x^4 - 2x^3 + 5x^2 \end{aligned}$$

answer these questions:

(a) Is $q \in \operatorname{Span}(p_1, p_2)$?
(b) Is $f \in \operatorname{Span}(p_1, p_2)$?

Prove your answers.

Problem 0.5. Suppose

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

and

$$\mathbf{y} = \begin{bmatrix} 1 \\ r \\ -1 \end{bmatrix}$$

for some real variable r. Find the value of r that makes the length of $\mathbf{x} + \mathbf{y}$ as small as possible. You may use calculus, or what you recall from graphing polynomials, but do not use the dot or cross product, even if you know what these are.

Problem 0.6. Number 15, page 123, but you need only verify axioms A1 and A3.

Problem 0.7. Suppose the professor in *Things that are/aren't vector spaces* tries to fix things by adjusting scalar multiplication to now be

$$r \left\uparrow \begin{matrix} a_1 & a_2 \\ a_6 & a_3 \\ a_5 & a_4 \end{matrix} \right\downarrow = \left\uparrow \begin{matrix} ra_6 & ra_1 \\ ra_5 & ra_2 \\ ra_4 & ra_3 \end{matrix} \right\downarrow$$

and she still wants us to use

$$\left\uparrow \begin{matrix} a_1 & a_2 \\ a_6 & a_3 \\ a_5 & a_4 \end{matrix} \right\downarrow + \left\uparrow \begin{matrix} b_1 & b_2 \\ b_6 & b_3 \\ b_5 & b_4 \end{matrix} \right\downarrow = \left\uparrow \begin{matrix} a_6 + b_6 & a_1 + b_1 \\ a_5 + b_5 & a_2 + b_2 \\ a_4 + b_4 & a_3 + b_3 \end{matrix} \right\downarrow .$$

Show that these operations also fail to make the set of vollters into a vector space. Specifically, show that A7 fails.