HOMEWORK 3

Problem 1. The formula

$$\det(A + B) = \det(A) + \det(B)$$

is almost always false.

(a) Find an example with two-by-two matrices where this formula is false.
(b) Find an example with two-by-two matrices where this formula is true.

Problem 2. Problem

Compute the determinant of A in two ways.

(a) Use elementary row operations to create an upper triangular matrix.
(b) Use expansion on the top row, at every stage, until you have the answer in terms of (many) two-by-two matrices that you evaluate with the "$ad - bc$" rule.

$$A = \begin{bmatrix} 3 & 3 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 \\ 1 & 2 & 5 & 6 \end{bmatrix}$$

Problem 3. If

$$A = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

find

$$\det(AB^{-1})$$

and

$$\det(B^{-1}A)$$

Problem 4. Suppose A is a 5-by-5 matrix that can be reduced to the identity by the row operations below, in the order given.

(a) What is $\det(A)$?
(b) What is the second column of A?
(c) What is

$$A \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} ?$$

Here are the row operations:

$$\begin{aligned}
&\mathrm{R4} - 2\mathrm{R2} \to \mathrm{R4} \\
&\mathrm{R4} \leftrightarrow \mathrm{R1} \\
&\frac{1}{3}\mathrm{R2} \to \mathrm{R2} \\
&\mathrm{R4} - 2\mathrm{R2} \to \mathrm{R4} \\
&\frac{1}{3}\mathrm{R2} \to \mathrm{R2} \\
&\mathrm{R1} - 2\mathrm{R3} \to \mathrm{R1} \\
&\frac{1}{6}\mathrm{R5} \to \mathrm{R5}
\end{aligned}$$

Problem 5. Define the matrix A as below for every value of r except 0. What is the determinant of A (in terms of r)?

$$A = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{r} \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1+r & \frac{2}{r} \\ 0 & 1 & 1-r & \frac{1-r}{r} \end{bmatrix}$$

Problem 6. Number 15 on page 81.

Problem 7. Number 5 on page 104.

Problem 8. Number 16 on page 104.