HOMEWORK 1 (REVISED)

Problem 1. Use back substitution to solve each of the following:

(a)

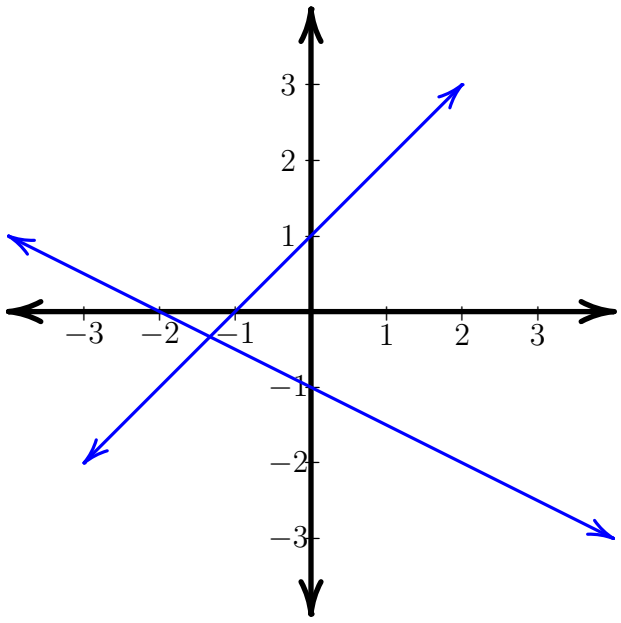
$$\begin{aligned} x + y + z &= 14 \\ y + 2z &= 6 \\ z &= 2 \end{aligned}$$

(b)

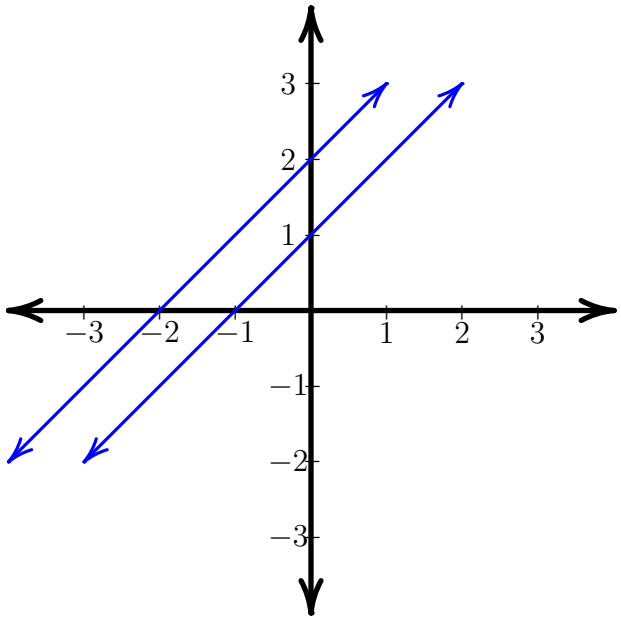
$$\begin{aligned} x_1 - x_2 + x_3 - x_4 &= 0 \\ x_2 + 3x_3 - 5x_4 &= 1 \\ x_3 + x_4 &= 0 \\ x_4 &= 2 \end{aligned}$$

Problem 2. For each figure, find a system of linear equations (in the standard form, as in §1.1) so that the solution sets to the individual equations are plotted.

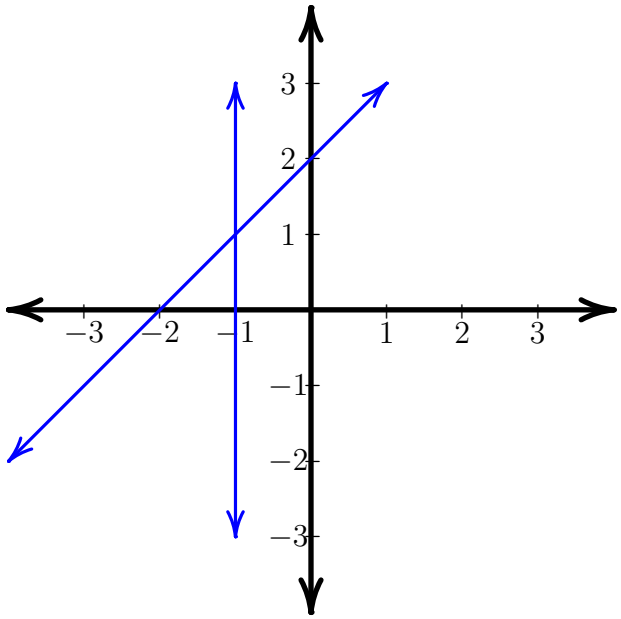
(a)



(b)



(c)



Problem 3. Mark the following matrices as to whether they are:

- not in row echelon form
- in row echelon form and not in reduced row echelon form
- in reduced row echelon form

Circle the pivots in any any matrix is in row echlon form.

(a)

$$\begin{bmatrix} -2 & 3 & 0 & 2 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 5 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 4. For each matrix listed, find a single *elementary* row operation that can be applied to yield a matrix that is in row echelon form. *This is the problem that changed.*

(a)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 4 \\ 1 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 5. Using only elementary row operations, transform the following matrix into row echelon form:

$$\begin{bmatrix} 3 & 6 & 3 & 12 \\ -2 & -2 & -2 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Problem 6. Find all solutions to the following system, using the associated augmented matrix and row operations leading to *reduced* row echelon form.

$$\begin{aligned} 2x_1 \qquad\quad &+ 4x_3 &&= 20 \\ x_1 + x_2 &+ x_3 &&= 9 \\ -x_1 + x_2 &+ x_3 &&= 5 \end{aligned}$$

Problem 7. Find all solutions to the following system.

$$\begin{aligned} x_1 + x_2 + x_3 &+ \qquad &+ \qquad &= 0 \\ x_1 + x_2 \qquad &+ 2x_4 + x_5 &&= 12 \\ + \qquad &+ 2x_3 - 4x_4 + x_5 &&= 6 \end{aligned}$$