MATH 316 Exam 1 — Spring 2022 — Review Topics and Problems

1. Solutions of first order ODEs $\frac{dy}{dt} = f(t, y)$

- Show that a given function solves a given ODE
  Examples: HW 1: 3
- Solve **separable** equations
  $$y' = f(t)g(y)$$
  using separation of variables. Careful!! Only separate variables if $g(y) \neq 0$. Also: be careful with your algebra, and always check your antiderivatives.
  Examples: 11th ed: §2.2 7,8,12,13. HW 1: 4. HW 2: 5,6,8. HW 3: 3,4,6,8,9
- Solve **linear** equations
  $$y' + p(t)y = q(t)$$
  using integrating factors.
  What is the main idea behind the method? Can you derive the integrating factor without memorizing formulas?
  Examples: 11th ed: §2.1: 12,14,21. HW 1: 6. HW 2: 1,2,3,5. HW 3: 5,9
- Investigate **autonomous** equations drawing the phase line and direction fields. Find equilibria and their stability.
  Examples: §1.1, §2.5: first set of problems. HW 2: 1-3,5. HW 3: 5,9
- Know: Integration by parts, partial fraction, substitution.
- Use the solutions you found (as well as direction fields and phaseline where available) to investigate limiting behaviour as $t \to \infty$, or as $t \to 0$, and the dependence of the limiting behaviour on the initial condition.
  Examples: §2.1: Example 3,4. HW 2: 1-3,7. HW 3: 5,7,8,9,10

2. Mathematical Models

- Find differential equation models for simple applications (mixing, population dynamics, falling objects with or without drag, raindrop, bucket)
  Examples: 10th ed: §1.1: 22,23,24,25. HW 2: 4-8, HW 3: 4-10

3. Theory

- What can you say about the existence and uniqueness of a solution to a linear first order ode, $y' + p(t)y = q(t)$?
- What can you say about the existence and uniqueness of a solution to a nonlinear first order ode, $y' = f(t, y)$?
  Examples: HW 2: 1-3,9,10. HW 3: 1-3,11
- Give an example of a nonlinear ODE whose solution blows up in finite time.
  Give an example of a linear ODE whose solution does not exist for all time.

4. Combo problem

Consider the temperature T of coffee in a cup in a room of ambient temperature T_a.

(a) Using Newton's law of cooling, find an (autonomous) ODE modeling the temperature T.
(b) Draw the phaseline, determine the stability of all equilibria, and draw several solution curves in the t-T plane.
(c) Find *all* the solutions to the ODE with initial condition $T(t_0) = T_0$, using the method of separation of variables.
(d) Find *all* the solutions to the ODE with initial condition $T(t_0) = T_0$, using the method of integrating factor.
(e) For how long does the temperatures function $T(t)$ exist, continuously? Explain, using both your above results and the theoretical results of §2.4.
(f) If the coffee in two cups in the same room have two different temperatures at time t_0, will they ever have the same temperature? Explain.