Lecture 18: Cauchy-Goursat

Theorem: Let $f = u+iv$ be analytic in a region D, C be a simple, closed, oriented curve contained in D. Then

$$\oint_C f(z)dz = 0$$

where the circle denotes that the integral is over a closed curve, with counterclockwise orientation.

Proof: in class, using Green's Theorem from Calc III

Example 1: By our earlier result, $\overline{z}$ is not analytic, since $\int_C \overline{z}\, dz \neq 0$ where C is unit circle centered at origin

Example 2: Same applies to $1/(z-z_0)$

Example 3: As we saw for one example, if C is a simple, closed curve, then $\int_C z^2\, dz = 0$

Note 1: It follows that if f is analytic, line integrals from A to B are path-independent.

Note 2: It also follows that if f is analytic in a region containing a hole, the integral over this region, with the correct orientation, is zero.