1. Exponential and logarithmic function

Let $f(z) = e^z$. Is f differentiable? Use C-R. What is f'(z)? Note: f is entire.

Let f(z) = Log(z). Is f differentiable? Use C-R in Cartesian coordinates. What is f'(z)? Repeat using C-R in polar coordinates. Where is f differentiable?

2. Harmonic functions

Definition: A function g(x, y) is harmonic if it satisfies Laplace's equation:

$$g_{xx} + g_{yy} = 0$$

Note: if f(z) = u(x, y) + iv(x, y) is analytic, then both u and v are harmonic (show it).

Definition: If f(z) = u + iv is analytic, then v is the harmonic conjugate of u.

Note: If v is the harmonic conjugate of u(f(z) = u(x, y) + iv(x, y) is analytic), then u is NOT the harmonic conjugate of v, unless both u, v are identically constant (show it).

Example 1: Let $u(x, y) = y^3 - 3x^2y$. Is *u* harmonic? (Check if $u_{xx} + u_{yy} = 0$. Answer: YES) Find the harmonic conjugate *v*. (Set up C-R equations for v_x and v_y . Find v(x, y) for the given ∇v .)