## Lecture 12: Exponents. Logs. Harmonic functions.

## 1. Exponential and logarithmic function

Let $f(z)=e^{z}$. Is $f$ differentiable? Use C-R. What is $f^{\prime}(z)$ ? Note: $f$ is entire.

Let $f(z)=\log (z)$. Is $f$ differentiable? Use C-R in Cartesian coordinates. What is $f^{\prime}(z)$ ? Repeat using C-R in polar coordinates. Where is $f$ differentiable?

## 2. Harmonic functions

Definition: A function $g(x, y)$ is harmonic if it satisfies Laplace's equation:

$$
g_{x x}+g_{y y}=0
$$

Note: if $f(z)=u(x, y)+i v(x, y)$ is analytic, then both $u$ and $v$ are harmonic (show it).

Definition: If $f(z)=u+i v$ is analytic, then $v$ is the harmonic conjugate of $u$.
Note: If $v$ is the harmonic conjugate of $u(f(z)=u(x, y)+i v(x, y)$ is analytic $)$, then $u$ is NOT the harmonic conjugate of $v$, unless both $u$, $v$ are identically constant (show it).

Example 1: Let $u(x, y)=y^{3}-3 x^{2} y$. Is $u$ harmonic? (Check if $u_{x x}+u_{y y}=0$. Answer: YES) Find the harmonic conjugate $v$. (Set up C-R equations for $v_{x}$ and $v_{y}$. Find $v(x, y)$ for the given $\nabla v$.)

