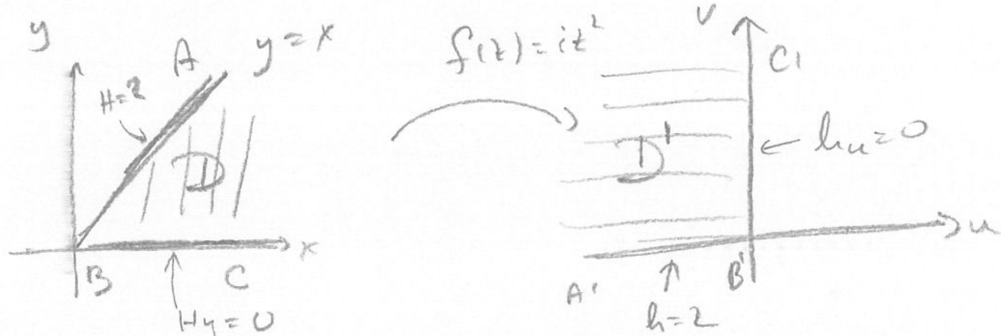
1. Find the linear fractional transformation that maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ onto the points $w_1 = 1$, $w_2 = i$, $w_3 = -1$.

2. Consider a generic curve going through the point $z_0 = 2 + i$ and its image under the transformation $w = z^2$. Determine the angle of rotation of the tangent vector to the curve and illustrate it for some particular curve.

3. (a) Show that the analytic function $f(z) = iz^2 = u(x, y) + iv(x, y)$ maps the half line $y = x, x > 0$, onto the negative u axis, the positive x axis conformally onto the positive v axis, and the region D in between to the 2nd quadrant D' (see figure).
   (b) Show that the function $h(u, v) = v + 2$ is harmonic in D' and satisfies the boundary conditions $h = 2$ on the negative u axis, and $\partial h/\partial u = 0$ on the positive v axis.
   (c) Conclude that the function $H(x, y) = h(u(x, y), v(x, y)$ is harmonic in D with boundary conditions $H = 2$ on the half-line $y = x$ and $\partial H/\partial y = 0$ on the positive x axis. Find $H(x, y)$.



4. (a) Show that the analytic function $f(z) = e^z = u(x, y) + iv(x, y)$ maps the line $0 \le y \le \pi, x = 0$, onto the semi-circular curve $u^2 + v^2 = 1$, $v \ge 0$, and the region D outside the line to region D' outside the circle (see figure).
   (b) Show that the function

$$h(u, v) = Re(2 - w + \frac{1}{w}) = 2 - u + \frac{u}{u^2 + v^2}$$

   is harmonic in D' and satisfies the boundary conditions $h = 2$ on the semicircle.
   (c) Conclude that the function $H(x, y) = h(u(x, y), v(x, y)$ is harmonic in D with boundary conditions $H = 2$ on the line-segment. Find $H(x, y)$.