1. Use residues to show that

   (a) $\displaystyle\int_0^\infty \frac{x^2\,dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$

   (b) $\displaystyle\int_0^\infty \frac{\cos(ax)\,dx}{x^2+1} = \frac{\pi}{2}e^{-a}$

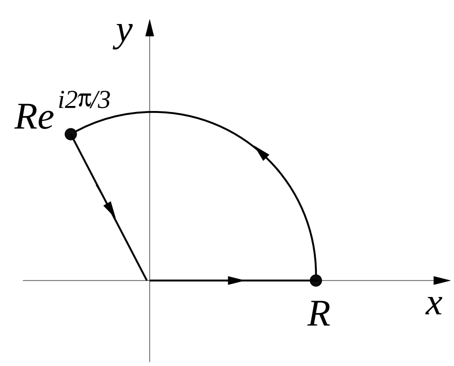
2. Use residues to evaluate the following integrals

   (a) $\displaystyle\int_0^\infty \frac{x^2\,dx}{(x^2+1)^2}$

   (b) $\displaystyle\int_0^\infty \frac{x\sin x\,dx}{(x^2+1)(x^2+4)}$

   (c) $\displaystyle\int_0^\infty \frac{x^3\sin x\,dx}{(x^2+1)(x^2+9)}$

3. Show that if $f(z) = p(z)/q(z)$, where p, q are analytic at z_0, and q has a simple root at z_0, then the residue of f at z_0 is
$$\frac{p(z_0)}{q'(z_0)}\ .$$



4. Use residues and the contour shown in the figure to establish the integration formula
$$\int_0^\infty \frac{dx}{x^3+1} = \frac{2\pi}{3\sqrt{3}}$$