

Reading : Schaum's example 6.26, and definition of singularities on page 175

1. Show that

$$(a) \csc z = \frac{1}{z} + \frac{1}{3!}z + \left[\frac{1}{(3!)^2} - \frac{1}{5!} \right] z^3 + \dots \text{ for } 0 < |z| < \pi$$

$$(b) \frac{1}{e^z - 1} = \frac{1}{z} - \frac{1}{2} + \frac{1}{12}z - \frac{1}{720}z^3 + \dots \text{ for } 0 < |z| < 2\pi$$

2. Expand each of the following functions in a Laurent series about $z = 0$, and name the type of the singularity in each case. Evaluate the integral of the function about the circle $|z| = 2$.

$$(a) f(z) = \frac{1 - \cos(z)}{z}$$

$$(d) f(z) = \frac{z^2}{e^{z^4}} = z^2 e^{-z^4}$$

$$(b) f(z) = \frac{e^z}{z^3}$$

$$(e) f(z) = \frac{\sin^2 z}{z}$$

$$(c) f(z) = \frac{1}{z \cosh z}$$

$$(f) f(z) = z e^{1/z^2}$$

3. Determine and classify all the singularities of the functions

$$(a) f(z) = \frac{1}{(2 \sin z - 1)^2}$$

$$(c) f(z) = \frac{z}{e^z - 1}$$

$$(b) f(z) = \frac{z}{e^{1/z} - 1}$$

$$(d) f(z) = \cos(z^2 + z^{-2})$$

4. Evaluate

$$(a) \oint_{|z|=3} \frac{z+1}{z^2 - 2z} dz$$

$$(c) \oint_{|z|=2} \frac{3z^3 + 2}{(z-1)(z^2+9)} dz$$

$$(b) \oint_{|z|=3} \frac{z+1}{z^3 - 2z^2} dz$$

$$(d) \oint_{|z|=4} \frac{3z^3 + 2}{(z-1)(z^2+9)} dz$$