1. (a) Evaluate $\int_{-3}^{3} \sqrt{z}\, dz$
   (b) Find the principal branchcut of $f(z) = \sqrt{z^2-1} = \sqrt{z-1}\sqrt{z+1}$.
   (c) Compute the integral $\int_{-i}^{i} \sqrt{z^2-1}\, dz$ using a path that follows the right side of the branch-cut of f. (Use the integration formula in Schaum's Outline, p116.) Note: The principal branch of the antiderivative function

$$F(z) = \frac{z}{2}\sqrt{z^2-1} - \frac{1}{2}\ln(z + \sqrt{z^2-1})$$

   is analytic everywhere except on the line $z = \alpha$, $\alpha \in (-\infty, 1)$. (Can you show this?)

2. (a) Let $C_R : |z - z_0| = R$, with counterclockwise orientation, and f be analytic in a region containing C_R. Show that $\frac{1}{2\pi i}\int_{C_R} \frac{f(z)}{z-z_0} dz$ is the average value of f over C_R.
   (b) List 2-3 conclusions you can draw from the above.

3. In class we showed that

$$\Big|\int_C f(z)\, dz\Big| \le \int_a^b \big|f(z(t))z'(t)\big|\, dt \le M \int_a^b |z'(t)|\, dt = ML$$

   where M is an upper bound for $|f|$ on C ($|f(z)| \le M$ for all $z \in C$) and $L = \int_a^b |z'(t)|\, dt$ is the length of C.
   (a) Use this fact to show that if f is analytic in $|z| \le 1$, and $C_\rho : |z| = \rho < 1$, then

$$\Big|\int_{C_\rho} z^{-1/2} f(z)\, dz\Big| \le 2\pi M\sqrt{\rho}$$

   for some M that is independent of ρ. Conclude that as $\rho \to 0$, the integral vanishes.
   (b) Suppose C_R is the upper semicircle $|z| = R$, with $\theta \in [0, \pi]$. Show that

$$\lim_{R\to\infty} \int_{C_R} \frac{2z^2-1}{z^4}\, dz = 0\ .$$

4. Let $C : |z - i| = 2$, with counterclockwise orientation. Evaluate
   (a) $\oint_C \frac{dz}{z^2+4}$ (b) $\oint_C \frac{dz}{(z^2+4)^2}$ (c) $\oint_C \frac{dz}{4z^2+1}$ (d) $\oint_C \frac{dz}{(4z^2+1)^2}$

5. Let $C : |z| = 3$, with counterclockwise orientation. Evaluate

$$g(z) = \oint_C \frac{2s^2 - s - 2}{s - z}\, ds$$

for z either in the circle, or outside.

6. Let C denote the boundary of the square $[-3,3]\times[-3,3]$, with counterclockwise orientation. Evaluate

(a) $\displaystyle\oint_C \frac{\tan(z/2)}{(z-x_0)^2}\,dz\ , \quad -3 < x_0 < 3$ (b) $\displaystyle\oint_C \frac{\cosh(z)}{z^4}\,dz$ (c) $\displaystyle\oint_C \frac{2s^4-s^2-2}{(s-z)^3}\,ds$

7. If a function f fails to be analytic at a point z_0, but is analytic at some point in every neighbourhood of z_0, then z_0 is called a *singular point* of f. Find all the singular points of
(a) $f(z) = \mathrm{sech} z$ (b) $f(z) = \tan z$

8. Let f be analytic in a region D. Let C be a generic closed simple smooth curve in D, positively oriented, enclosing three distinct points z_1, z_2, z_3. Let $C_k : |z - z_k| = \epsilon_k$, $k = 1, 2, 3$, be three circles, all positively oriented, and so small that they are completely contained in C and do not intersect. Let

$$g(z) = \frac{f(z)}{(z-z_1)(z-z_2)^2(z-z_3)^3}\ .$$

Show that

$$\oint_C g(z)dz = \oint_{C_1} g(z)dz + \oint_{C_2} g(z)dz + \oint_{C_3} g(z)dz\ .$$