1. Find the real and imaginary parts of $f(z) = \sin(z)$.

2. (a) Suppose $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y)\rangle$ is a vector field defined in $\Re^2$. State Green's Theorem.
   (b) Verify Green's Theorem for $\mathbf{F} = \langle x, -y\rangle$ where $C$ is the closed curve surrounding the square $[-1, 1] \times [-1, 1]$, with counterclockwise orientation. (Green's theorem equates a line integral and an area integral. Verifying for an example means that you compute both and confirm that they are equal for the particular example.)
   (c) Verify Green's Theorem for $\mathbf{F} = \langle x^4, xy\rangle$ where $C$ is the closed curve surrounding the triangle with corners (0,0) (1,0) (0,1), with counterclockwise orientation.
   (d) Use Green's Theorem to evaluate

$$\oint_C (3y - e^{\sin x})\, dx + (7x + \sqrt{y^4 + 1})\, dy$$

   where $C$ is the circle $x^2 + y^2 = 9$, with counterclockwise orientation.

3. In class we showed that $|\int_C f(z)\, dz| \le \int_a^b |f(z(t))z'(t)|\, dt \le M \int_a^b |z'(t)|\, dt$ where $M$ is an upper bound for $|f|$ on C, that is, $|f(z)| \le M$ for all $z \in C$.
   (a) Use this fact to show that

$$\Big| \int_{C_R} \frac{\mathrm{Log} z}{z^2}\, dz \Big| < 2\pi \Big( \frac{\pi + \mathrm{Log} R}{R} \Big)$$

   where $C_R$ is the circle $|z| = R > 1$, with counterclockwise rotation.
   (b) Use your result in (a) to show that the limit

$$\lim_{R\to\infty} \int_{C_R} \frac{\mathrm{Log} z}{z^2}\, dz = 0\ .$$

4. Evaluate $\int_C f(z)\, dz$ where $f(z) = y - x + i3x^2$ and $C$
   (a) is the line segment from $z = 0$ to $z = 1 + i$
   (b) consists of two line segments, one from $z = 0$ to $z = i$ and the other from $z = i$ to $z = 1+i$.
   (c) Is $f$ analytic in a region containing $C$? Explain.

5. Evaluate $\int_C f(z)\, dz$ where $f(z) = \frac{z+2}{z}$ and $C$ is
   (a) the semicircle $z = 2e^{i\theta}$, $\theta \in [0, \pi]$
   (b) the semicircle $z = 2e^{i\theta}$, $\theta \in [\pi, 2\pi]$
   (c) the semicircle $z = 2e^{i\theta}$, $\theta \in [0, 2\pi]$
   (d) Is $f$ analytic in a region containing $C$? Explain.

6. Evaluate the following line integrals

   (a) $\oint_C (3z+1)\,dz$ where $C$ is the boundary of the square with vertices at the points $z = 0, 1, 1+i, i$, with counterclockwise orientation.

   (b) $\oint_C \pi e^{\pi \overline{z}}\,dz$ where $C$ is as in (a).

   (c) $\int_C z^{1/2}\,dz$ were $C$ is the semicircular path $z = e^{i\theta}$, $\theta \in [0, \pi]$, from $z = 1$ to $z = -1$.

   (d) $\oint_C \frac{z^2}{z-3}\,dz$ were $C$ is the circle $|z| = 1$, with counterclockwise orientation.

   (e) $\oint_C \operatorname{sech} z\,dz$ were $C$ is the circle $|z| = 1$, with counterclockwise orientation.

   (f) $\oint_C \tan z\,dz$ were $C$ is the circle $|z| = 1$, with counterclockwise orientation.

   (g) $\oint_C \operatorname{Log}(z+2)\,dz$ were $C$ is the circle $|z| = 1$, with counterclockwise orientation.

7. Let $C$ denote the boundary of the square $[-4, 4] \times [-4, 4]$, with counterclockwise orientation. Evaluate

   (a) $\oint_C \frac{e^{-z}dz}{z - (\pi i/2)}$

   (b) $\oint_C \frac{\cos z}{z(z^2+8)}\,dz$

   (c) $\oint_C \frac{z\,dz}{2z+1}$

   (d) $\oint_C \frac{1}{(s-z)^3}\,ds$