MATH 313 — HOMEWORK # 1 Due : Friday, Jan 29, 2016.

TOPICS COVERED

Complex numbers

Cartesian form $z = a + ib$
Algebra in cartesian form. Geometric representation
Modulus, conjugate. Properties. Triangle inequality
Polar form $z = r(\cos\theta + i\sin\theta)$
Euler's formula, exponential form $z = re^{i\theta}$
Algebra in exponential form (products, quotients, powers, roots)
Fundamental Theorem of Algebra

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1. Compute $\dfrac{5}{(1-i)(2-i)(3-i)}$.

2. Prove that $\overline{z_1 z_2} = \overline{z_1}\ \overline{z_2}$

3. Prove that $|z_1 z_2| = |z_1||z_2|$

4. Show that $\big||z_1| - |z_2|\big| \leq |z_1 + z_2|$ Hint: start from $z_1 = z_1 + z_2 - z_2 = z_1 + z_2 + (-z_2)$ and use the triangle inequality. Repeat with z_2.

5. Use complex numbers to show that the diagonals of a parallelogram bisect each other.

6. Let $z_1 = 1 + i$, $z_2 = -1 + \sqrt{3}i$. For each of the following, find the cartesian and the exponential form, and state $Arg(z)$, $|z|$, $Re(z)$, $Im(z)$.
   (a) z_1 and z_2 , (b) z_1^{10} , (c) $z_1 z_2$, (d) z_1/z_2 .

7. Find each of the following complex numbers z in cartesian form by first writing the individual factors in exponential form, performing the needed operations, and finally changing back to cartesian coordinates. State $Arg(z)$, $|z|$, $Re(z)$, $Im(z)$.
   (a) $z = \dfrac{-2}{1+\sqrt{3}i}$, (b) $z = \dfrac{i}{-2-2i}$, (c) $z = (\sqrt{3} - i)^6$,
   (d) $z = i(1 - \sqrt{3}i)(\sqrt{3} + i)$, (e) $z = i(1 + \sqrt{3}i)^{-10}$.

8. Show that (a) $|e^{i\theta}| = 1$, (b) $\overline{e^{i\theta}} = e^{-i\theta}$, (c) $(e^{i\theta})^2 = e^{2i\theta}$,

9. (a) Use 8(c) above to show that
   $$\sin(2\theta) = 2\sin(\theta)\cos(\theta)\ , \quad \cos(2\theta) = \cos^2\theta - \sin^2\theta)\ .$$
   (b) Use $\cos^2\theta + \sin^2\theta = 1$ and your result in (a) to show that
   $$\cos^2\theta = \frac{1+\cos 2\theta}{2}\ , \quad \sin^2\theta = \frac{1-\cos 2\theta}{2}\ .$$

10. In each case, find all the roots, exhibit them geometrically, and point out which is the principal root. Note that the roots $z = (z_0)^{1/n}$ are all the n roots of the equation $z^n = z_0$.
    (a) $(2i)^{1/2}$, (b) $(1 - \sqrt{3}i)^{1/2}$, (c) $(-1)^{1/3}$,
    (d) $(-16)^{1/4}$, (e) $8^{1/6}$, (f) $(-4\sqrt{2} + 4\sqrt{2}i)^{1/3}$.