MATH 313 Exam 3— Review Topics and Problems

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1. Finding Taylor series and Laurent series

- Using the definition (for Taylor series only) HW 8: 1,2,4.
- Using known series expansions, together with substitution and multiplication. HW 8: 3, 5, 6, 7, 8a, 9a. HW 9: 1,2,3. HW 10: 1, 2.
- Why are zeros of (nontrivial) analytic functions isolated?
- What is the domain of convergence of Taylor and Laurent series? HW 8: 4, 5.

2. Singular points and residues

- Classify singular points (poles of finite order, essential singularities, removable singularities). HW 10: 2, 3.
- Find residues using any of the methods summarized below.
  (1) Find the Laurent Series, at least a few terms thereof.
  (2) Using the Cauchy Integral formula $\phi(z_0) = \frac{1}{2\pi i}\oint_C \frac{\phi(z)}{z-z_0} ds$ where C surrounds z and f is analytic, in the following way:
  $$\oint_C \frac{\phi(z)}{(z-z_0)^{n+1}}\,dz = \frac{2\pi i}{n!}\phi^{(n)}(z_0)$$
  so the residue of f at z_0 is $b_1 = \phi^{(n)}(z_0)/n!$.
  (3) If $f(z) = p(z)/q(z)$ has a simple pole at z_0, where p, q are analytic at z_0, then the residue at z_0 is
  $$\frac{p(z_0)}{q'(z_0)}$$
  (Can you show this?) HW 11, #3
- Evaluate contour integrals using residues. HW 8: 8b, 9b. HW 10: 4. Also used in HW 11: 1, 2, 4.

3. Evaluating real integrals using residues

- Integrals of the form $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx$. HW 11: 1a, 2a, 4.
- Integrals of the form $\displaystyle\int_{-\infty}^{\infty} f(x)\cos x\,dx$ and $\displaystyle\int_{-\infty}^{\infty} f(x)\sin x\,dx$. HW 11: 1b, 2bc.
  Jordan's inequality. When do you need it? HW 11: 2c.