TOPICS COVERED  (Lectures 1-10)

1. Inverses (defining properties, graphs, derivatives)

2. Exponential, Logarithmic, Inverse Trigonometric, Hyperbolic functions
   - graphs, domains, ranges, values at particular x’s, whether even or odd
   - limiting values
   - derivatives

3. Integration methods
   - substitution
   - integration by parts
   - trigonometric integrals
   - trigonometric substitution
   - simple problems that don’t require any of the above

4. L’Hôpital’s rule and Relative Rates of Growth
   - compute limits with or without L’Hôpital’s rule, as appropriate
   - compute limits of improper forms s.a. ∞ − ∞, ∞ · 0, 0^0, ∞^0.
   - compare rates of growth of functions as x → ∞.

STUDY PROBLEMS

The exam is based on the problems like those in the homework, including
the mixed review problems. Here is a sample list of such problems.

0. Chapter 6, Review: Concept Check. True-False Quiz.

1. Use derivatives and limits to obtain graphs and extremal values of functions, as in
   - Day 2 Homework problem 7, include graph.
   - Day 5 Homework problem 4.
   - Day 6 Homework problems 1,2,3
   - Mixed Review R3.2, R5.1

2. Sketch graphs of exponentials, logarithms, inverse trig functions, translations thereof. Know
   domains, limiting values.

3. Find the equation for the tangent line to f^{-1}(x) at x = a.
   - (a) f(x) = 3 + x^2 + \tan(\pi x/2), a = 3
   - (b) f(x) = \int_{a}^{x} \sqrt{1 + t^3} dt, a = 0

4. Order the functions given below so that f_1 grows slower than f_2 grows slower than f_3 etc.
   - Write your answer using the following notation f_1 \ll f_2, f_3 \ll f_4 \ldots \ll f_n.
   - e^x, x^5, 2^x, x, x^2, x^3, x^{10000}, \sqrt[10]{x}, 1^x, 1.0000001^x, x^x, \ln(x), \ln(x^2), e^{2x}, \ln(2x)

5. Find the following limits
\begin{align*}
(a) \ &\lim_{x\to 0} \frac{\sin^{-1} x}{x} \quad (b) \ &\lim_{x\to 0^-} \tan^{-1} \left( \frac{\pi}{x} \right) \\
(c) \ &\lim_{x\to -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
(d) \ &\lim_{x\to \infty} \frac{1 + 2^x}{1 - 2^x} \\
(e) \ &\lim_{n\to \infty} (1 + \frac{r}{n})^n \\
(f) \ &\lim_{x\to 0^+} x^x
\end{align*}

6. Find the derivatives of the following functions

(a) \( f(x) = x^2 \) \quad (b) \( f(x) = x^2 \sqrt{x + 2} \) \quad (c) \( g(x) = \sqrt{x} \)

7. For what \( c \) is -1/2 the average value of \((x - c) \sin(x)\) over the interval \([0, \pi/3]\)?

8. Show that the function \( y = Ae^{-x} + Bxe^{-x} \) satisfies the differential equation \( y'' + 2y' + y = 0 \)

9. Compute an integral to show that the area of a circle of radius \( R \) is \( \pi R^2 \).

10. Evaluate the following indefinite integrals.

(a) \( \int \sqrt{2r - 1} \, dr \) \quad (b) \( \int \frac{1}{2 - 3s} \, ds \) \quad (c) \( \int \frac{e^{5x}}{5^{2x}} \, dx \)

(d) \( \int_0^1 \frac{r^3}{\sqrt{4 + r^2}} \, dr \) \quad (e) \( \int_1^2 \frac{\cos(1/r)}{r^2} \, dr \) \quad (f) \( \int \ln(x) \, dx \)

(g) \( \int x \sin(3x) \, dx \) \quad (h) \( \int x \sin(3x^2) \, dx \) \quad (i) \( \int \sin^2(3x) \, dx \)

(j) \( \int \sin(2x) \cos(3x) \, dx \) \quad (k) \( \int \frac{dx}{x^2 + 4} \) \quad (l) \( \int \frac{dx}{x^2 + 5} \)

(m) \( \int \frac{\sqrt{x^2 - 9}}{x} \, dx \) \quad (u) \( \int e^{\sqrt{t}} \, dt \)

11. Evaluate the following definite integrals.

(a) \( \int_0^t (t - s)^2 \, ds \) \quad (b) \( \int_0^t e^s \sin(t - s) \, ds \) \quad (c) \( \int_0^4 \frac{dt}{16 + t^2} \)

(d) \( \int_2^5 \frac{dr}{1 + 2r} \) \quad (e) \( \int_0^1 \frac{e^x}{1 + e^{2x}} \, dx \) \quad (f) \( \int_0^{\pi/2} \frac{\cos(x)}{1 + \sin^2 x} \, dx \)

(g) \( \int_0^{\pi/2} \sin^3(\theta) \cos^2(\theta) \, d\theta \) \quad (h) \( \int_1^2 \frac{(x + 1)^2}{x} \, dx \) \quad (i) \( \int_1^2 \frac{x}{(x + 1)^2} \, dx \)

(j) \( \int_1^2 x^5 \ln x \, dx \) \quad (k) \( \int_1^{\sqrt{3}} \frac{x^3 \, dx}{\sqrt{x^2 + 4}} \) \quad (l) \( \int_0^a x^2 \sqrt{a^2 - x^2} \, dx \)

12. If a water wave with length \( L \) moves with velocity \( v \) in a body of water with depth \( d \), then

\[ v = \sqrt{\frac{gL}{2\pi} \tanh \left( \frac{2\pi d}{L} \right)} \]

where \( g \) is acceleration due to gravity. Explain why the approximation

\[ v \approx \sqrt{\frac{gL}{2\pi}} \]

is appropriate in deep water.

13. §6.7: 51,52