0. Review Material
- Basic algebra
- Graph basic functions and translations and transformations
- Solve equalities and inequalities (algebraically and graphically)
   In the above, include trig functions and absolute value function.

0.1 Sketch the graphs of the following functions (for g-h, use superposition of elementary graphs)
(a) \( f(x) = \sin(3x) \) (b) \( g(t) = \frac{(1 - \cos(\pi t))}{2} \) (c) \( f(t) = 1 - e^{-t} \)
(d) \( f(t) = 3\sin(2\pi t/24) + 8 \) (e) \( f(t) = 2\tan t \) (f) \( f(t) = \sec t \)
(g) \( f(t) = \frac{1}{x} + x \) (h) \( f(t) = \frac{1}{x} + x^2 \) (i) \( f(t) = \frac{1}{x^2} + x^2 \)
(j) \( v(r) = r^3 - r_o r \) (k) \( f(x) = x^2(x^2 - a^2) \) (l) \( f(x) = |x + 1|/(x + 1) \)
(m) \( f(x) = |x^2 - 4|/(x - 2) \)

0.2 Solve the following inequalities and equalities (Hint: factor first. Graphs often help)
(a) \(|4x - 3| < 2\) (b) \(|x + 2| > 3\) (c) \(x^2 < 2\)
(d) \(x^3 - x < 0\) (e) \(x^3 - x^2 > 0\) (f) \(x^{1/3}(x + 4) > 0\)
(g) \(x^{1/2}(x + 4) > 0\) (h) \(\frac{1}{2}x^{-2/3}(x + 4) + x^{1/3} > 0\) (i) \(\frac{1}{3}x^{-5/3}(x + 4) + \frac{2}{5}x^{-2/3} < 0\)
(j) \(\tan x > 1, x \in (-\frac{\pi}{2}, \frac{\pi}{2})\) (k) \(\sin x > 1/2, x \in [0, 2\pi]\) (l) \(\sin(\pi x) > 1/2, x \in [0, 2]\)
(m) \(e^x > 0\) (n) \(\frac{x - 1}{e^{x-1}} = 0\) (o) \(\frac{1}{e^{x-1}} = 0\)

1. Rates of change
- Interpreting derivative as instantaneous rates of change, and find its units

1.1 The total cost (in $) of repaying a student loan at an interest rate of r% per year is \(C = f(r)\).
   (a) What is the meaning of the derivative \(f'(r)\)? What are its units?
   (b) What does the statement \(f'(10) = 1200\) mean?
   (c) Is \(f'(r)\) always positive or does it change sign?

1.2 A rod of length \(L\) is shown in the figure below. Suppose the mass of the rod between its left endpoint and a point at distance \(x\) from it is \(m(x), 0 \leq x \leq L\), where \(x\) is measured in cm and \(m\) is measured in kilograms. In one sentence, state the meaning of the derivative \(m'(x)\). What are its units?

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This part of the rod has mass \(m(x)\)

1.3 A particle moves on a straight line according to the law of motion \(s = t^3 - 9t^2 + 15t + 10, t \geq 0\), where \(t\) is measured in seconds and \(s\) measures the distance in feet of the particle from a fixed point \(O\).
   (a) Find the velocity at time \(t\).
   (b) When is the particle at rest?
   (c) When is the particle moving in the negative direction?
   (d) When is the particle moving in the positive direction?
   (e) Find the total distance traveled during the first 8 s.

It may help to draw a diagram that illustrates the motion of the particle as in Figure 2, page 165.

1.4 What is acceleration when velocity is constant?

1.5 §2.1: 49 (derivative of \(T(t)\))
1.7 The cost, in dollars, of producing $x$ yards of a certain fabric is $C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$.
   (a) What is the meaning of the derivative $C'(x)$?
   (b) Find $C'(200)$. What does it predict?
   (c) Compare $C'(200)$ with the cost of manufacturing the 2012st yard of fabric.

2. Related rates of change
   - Given relation between dependent variables, find a relation between their derivatives
   - Given a rate of change of one variable, find rate of change of another.
   Need to find relation between variables first.

2.1 The pressure $P$, temperature $T$ and volume $V$ of a gas are related by the rule $T = cPV$ where $c$ is a constant. At one instant the pressure of a gas is 30 KPa, the temperature is 18°C, and the volume is 152 mL. If at that instant the temperature is increasing at 1°C/hour and the volume is decreasing by 2mL/hour, at what rate is the pressure changing at that instant?

2.2 A rectangle is such that its length increases at the rate of 5 in/min and its width decreases at the rate of 7 in/min. At the moment when the length is 6 in and the width is 8 in, is the area increasing or decreasing?

2.3 §2.8: 16 (shadow)

2.4 §2.8: 26 (swimming pool)

2.5 §2.8: 35 (resistors)

3. Linearization
   - Approximating functions by linearization
   - Approximating changes of functions by change in linearization

3. §2.9: 5,7,31,35,38,40a,41

4. Finding Absolute Max/Min
   - Find abs max/min of continuous functions on closed intervals: find critical numbers and compare function values at critical numbers and at endpoints
   - Find abs max/min in general: find critical numbers and get enough information on increasing/decreasing or concavity to obtain a rough sketch of a graph that clearly illustrates absolute max/min.

4.1 Chapter 3 Review: Exercises 2, 4

4.2 Find the absolute maximum value and absolute minimum value of $f(x) = \cos^2 x - 2\sin x$ for $x \in [0, 2\pi]$.

4.3 §3.7: 9 (maximizing yield)

4.4 Chapter 3 Review: Exercise 45 (length of wave with smallest velocity)

4.5 §3.1: 67 (Find max of $v(r) = k(r_0 - r)r^2$)

4.6 §3.7: 43

5. Graphing
   - Use intervals of increase/decrease, concave up/down, limiting behaviour near vertical asymptotes, horizontal asymptotes, intercepts, domain, symmetry to sketch graph of function
5.1 Find domain, intervals where function is increasing/decreasing, intervals where function is concave up/down, and use behaviour at infinity if applicable, to obtain a good graph of the following functions. Clearly indicate the coordinates of all extrema and inflection points.

(a) \( f(x) = x^4 + 2x^3 + 2 \)  
(b) \( f(x) = x - x^3 \)  
(c) \( f(x) = x\sqrt{6 - x} \)  
(d) \( f(x) = \frac{ax}{x^2 + b^2}, a, b > 0 \)  
(e) \( f(x) = ax + \frac{b}{x^2}, a, b > 0 \)  
(f) \( f(x) = ax^2 + \frac{b}{x}, a, b > 0 \)  
(g) \( f(x) = x^{1/3}(x + 4) \)  
(h) \( g(x) = xe^{-x} \)  
(i) \( h(x) = \frac{e^x}{x^2} \)

5.2 Sketch a clear graph of the function in problem §2.7: 18, clearly indicating coordinates of all intercepts and extrema.

5.3 §3.5: 43 (Hint: simplify the expression for the function by replacing \( W/(24EI) \) by another letter, for example \( A \).)

6. Optimization

These are word problems in which you need to identify the function that is to be maximized or minimized and its domain, and then use calculus to find the absolute max or min of that function on the particular domain. Please follow the outline:

Step 1: Draw picture. Label all variables, and all constants.
Step 2: What is given? What do you want to maximize or minimize?
Step 3: Find a formula for the function you want to max/min. If more than one variable is involved, use what is known to reduce the formula to a function of one variable.
Step 4: Use any method you wish to find the absolute max or min. If the function is continuous on a closed interval, only need values at critical points and endpoints. Otherwise, get enough information using first and/or second derivatives to obtain a rough graph of the function that clearly illustrates its absolute max/min.
Step 5: Answer the question with a sentence (in correct English).

6.1 §3.7: 14, 20, 37

6.2 Assume that if the price of a certain book is \( p \) dollars, then it will sell \( x \) copies where \( x = 7000(1 - p/35) \). Suppose the dollar cost of producing those \( x \) copies is \( 15000 + 2.5x \). Finally, assume that the company will not sell this book for more than \$35. Determine the price for the book that will maximize profit.

7. Antiderivatives and differential equations

- Find antiderivatives of \( f(\xi) \), also written as \( \int f(\xi) \, d\xi \).
- Solve differential equations of form \( y' = f(x) \), or \( y'' = f(x) \), w/ or w/o initial conditions.
- Applications:
  
  Given acceleration, find velocity, distance travelled.
  
  (Don’t use memorized formulas you may have seen in physics.)
  
  Given linear density, find mass.
  
  Given a rate of change of a quantity, find information about the quantity.

7.1 §3.9: 5, 13 (find antiderivatives)
7.2 §3.9: 33 (solve differential equation)
7.3 §3.9: 60,67 (applications)
7.4 §6.2: 97,98 (applications)
7.5 §6.2: 55 (show \( y \) satisfies differential equations)