In[301]:=
```
n = 3;
```

In[302]:=
```
sigmax = {{0, 1}, {1, 0}};
```

In[303]:=
```
sigmay = {{0, -i}, {i, 0}};
```

In[304]:=
```
sigmaz = {{1, 0}, {0, -1}};
```

In[305]:=
```
I2 = IdentityMatrix[2];
```

In[306]:=
```
u = DiagonalMatrix[ConstantArray[1, n - 1], 1] +
    DiagonalMatrix[ConstantArray[1, 1], -n + 1];
```

In[307]:=
```
v = DiagonalMatrix[Exp[2 Pi I / n]^Range[n]];
```

In[308]:=
```
AA = (1/2) * (ConjugateTranspose[u] + u);
```

In[309]:=
```
BB = (i/2) * (ConjugateTranspose[u] - u);
```

In[310]:=
```
CC = (1/2) * (ConjugateTranspose[v] + v);
```

In[311]:=
```
DD = (i/2) * (ConjugateTranspose[v] - v);
```

In[312]:=
```
loclzr = KroneckerProduct[i * sigmax, AA - w * IdentityMatrix[n]] +
    KroneckerProduct[i * sigmay, BB - x * IdentityMatrix[n]] +
    KroneckerProduct[i * sigmaz, CC - y * IdentityMatrix[n]] +
    KroneckerProduct[I2, DD - z * IdentityMatrix[n]];
```

In[313]:=
```
charpoly = Det[loclzr];
```

In[314]:=
```
impoly = FullSimplify[Im[charpoly],
  {Element[w, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals]}]
```

Out[314]=
$$\frac{3}{2}\sqrt{3}\ \left(w^2 + x^2 - y^2 - z^2\right)$$

In[315]:=
```
realpoly = FullSimplify[Re[charpoly], {w^2 + x^2 == y^2 + z^2,
    Element[w, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals]}];
```

In[316]:=
```
realpoly = ReplaceAll[realpoly, {x → Sqrt[-w^2 + y^2 + z^2]}];
```

In[317]:=
```
realpoly =
 FullSimplify[realpoly, {Element[w, Reals], Element[y, Reals], Element[z, Reals]}]
```

Out[317]=
$$-1 - 8\,w^3 + 3\,y^2 + 6\,w\,y^2 - 2\,y^3 + 12\,y^4 + 8\,y^6 + 3\left(1 + 2\,w + 2\,y + 8\,y\left(y + y^3\right)\right)z^2 + 12\left(1 + 2\,y^2\right)z^4 + 8\,z^6$$

In[318]:=
```
altpoly = ReplaceAll[realpoly, {w → r * Cos[th], y → r * Cos[phi], z → r * Sin[phi]}];
```

In[319]:=
```
altpoly = FullSimplify[altpoly, {r > 0, Element[th, Reals], Element[phi, Reals]}];
```

In[320]:=
```
Collect[ExpandAll[altpoly], r]
```

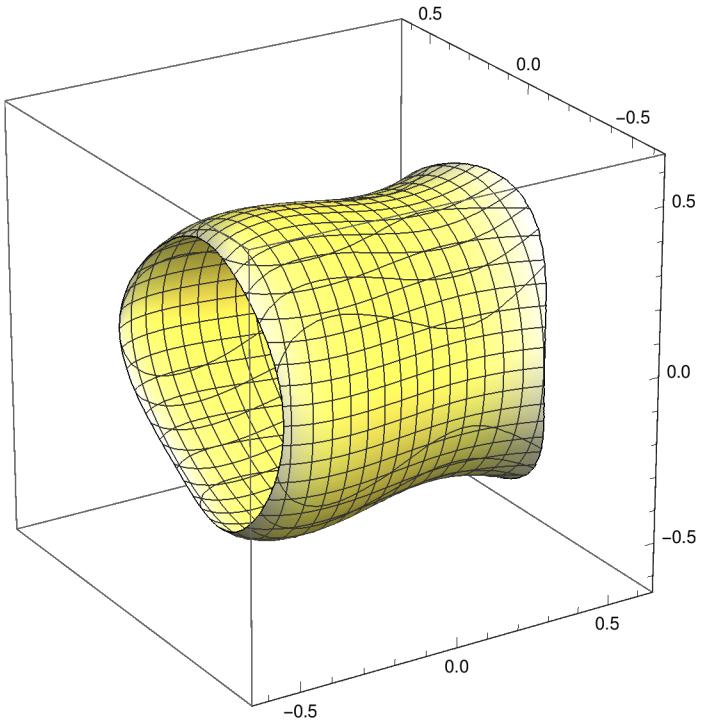
Out[320]=
$$-1 + 3\,r^2 + 12\,r^4 + 8\,r^6 + r^3\left(-2\,\text{Cos}[3\,\text{phi}] - 2\,\text{Cos}[3\,\text{th}]\right)$$

In[321]:=
```
Collect[Expand[D[altpoly, r]], r]
```

Out[321]=
$$6\,r + 48\,r^3 + 48\,r^5 + r^2\left(-6\,\text{Cos}[3\,\text{phi}] - 6\,\text{Cos}[3\,\text{th}]\right)$$

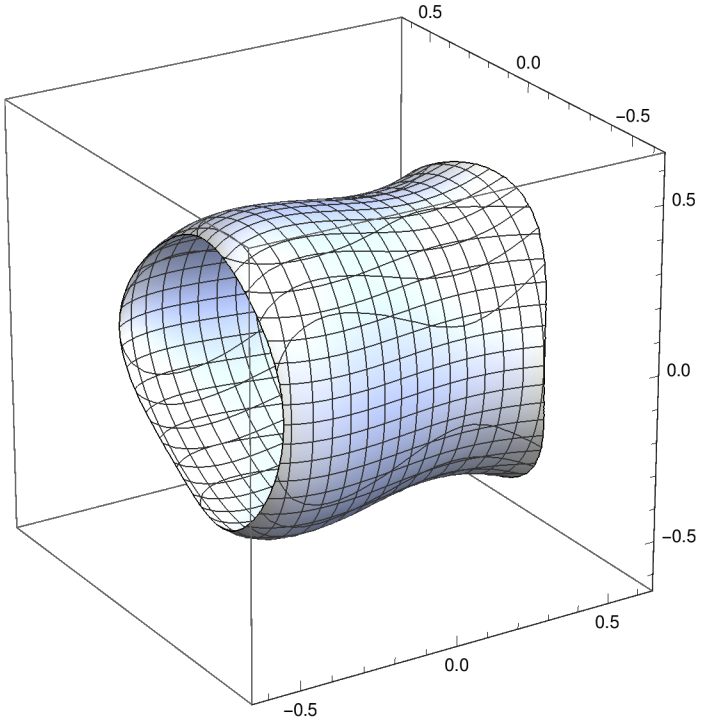
In[322]:=
```
upPlot = ContourPlot3D[realpoly == 0, {w, -5/8, 5/8}, {y, -5/8, 5/8},
  {z, -5/8, 5/8}, RegionFunction → Function[{w, y, z}, 0 < y^2 + z^2 - w^2],
  ColorFunction → Function[{w, y, z},
    ColorData["TemperatureMap"][0.5 + 0.5 Sqrt[y^2 + z^2 - w^2]]],
  ColorFunctionScaling → False, ViewPoint → {-5, -8, 4},
  BoxRatios → Automatic, PlotPoints → 25, MaxRecursion → 3]
```

Out[322]=



In[323]:=
```
downPlot = ContourPlot3D[realpoly == 0, {w, -5/8, 5/8}, {y, -5/8, 5/8},
  {z, -5/8, 5/8}, RegionFunction → Function[{w, y, z}, 0 < y^2 + z^2 - w^2],
  ColorFunction → Function[{w, y, z},
    ColorData["TemperatureMap"][0.5 - 0.5 * Sqrt[y^2 + z^2 - w^2]]],
  ColorFunctionScaling → False, ViewPoint → {-5, -8, 4},
  BoxRatios → Automatic, PlotPoints → 25, MaxRecursion → 3]
```

Out[323]=



In[324]:=
```
Export["torus_4matrices_n3_up.eps", upPlot, ImageSize → 3.2 * 72];
```

In[325]:=
```
Export["torus_4matrices_n3_down.eps", downPlot, ImageSize → 3.2 * 72];
```