```
In[•]:= (* Testing with odd and three  even. *)

In[•]:= n = 1;

In[•]:= sigmax = {{0, 1}, {1, 0}};

In[•]:= sigmay = {{0, -i}, {i, 0}};

In[•]:= sigmaz = {{1, 0}, {0, -1}};

In[•]:= I2 = IdentityMatrix[2];

In[•]:= AA = 2 * KroneckerProduct[sigmaz, sigmax];

In[•]:= MatrixForm[AA]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & 0 \end{pmatrix}$$

```
In[•]:= BB = {{0, -i, 0, 0}, {i, 0, 0, 0}, {0, 0, 0, i}, {0, 0, -i, 0}};

In[•]:= MatrixForm[BB]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}$$

```
In[•]:= CC = KroneckerProduct[sigmaz, sigmaz];

In[•]:= MatrixForm[CC]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[•]:= DD =  {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}};

In[•]:= MatrixForm[DD]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

```
In[•]:= loclzrHalf = KroneckerProduct[i * sigmax, AA -  w * IdentityMatrix[4]] +
          KroneckerProduct[i * sigmay, BB - x * IdentityMatrix[4]] +
          KroneckerProduct[i * sigmaz, CC -  y * IdentityMatrix[4]] +
          KroneckerProduct[I2, DD -  z * IdentityMatrix[4]];

In[•]:= charpoly = FullSimplify[Det[loclzrHalf]]
```

Out[•]=

$$\left(w^4 + 2\,w^2\,\left(-3 + x^2 + y^2 + z^2\right) + \left(3 + x^2 + y^2 + z^2\right)^2\right)$$
$$\left(w^4 + 2\,w^2\,\left(1 + x^2 + y^2 + z^2\right) + \left(-1 + x^2 + y^2 + z^2\right)\,\left(15 + x^2 + y^2 + z^2\right)\right)$$

```
In[•]:= charpolyReduce = ReplaceAll[charpoly, {x^2 + y^2 + z^2 → R^2}]
```

$$\text{Out[•]=}\ \left(\left(3+R^2\right)^2+2\left(-3+R^2\right)w^2+w^4\right)\left(\left(-1+R^2\right)\left(15+R^2\right)+2\left(1+R^2\right)w^2+w^4\right)$$

```
In[•]:= charpolyReduce = Factor[charpolyReduce]
```

$$\text{Out[•]=}\ \left(9+6R^2+R^4-6w^2+2R^2w^2+w^4\right)\left(-15+14R^2+R^4+2w^2+2R^2w^2+w^4\right)$$

```
In[•]:= charpolyReduceRotate =
         FullSimplify[ReplaceAll[charpolyReduce, {R^2 → x^2 + y^2 + z^2}]]
```

$$\text{Out[•]=}\ \left(9+R^4+w^4+6x^2+6y^2+6z^2+2w^2\left(-3+x^2+y^2+z^2\right)\right)$$
$$\left(-15+R^4+2w^2+w^4+14\left(x^2+y^2+z^2\right)+2w^2\left(x^2+y^2+z^2\right)\right)$$

```
In[•]:= shouldBeZero = FullSimplify[charpolyReduceRotate - charpoly]
```

$$\text{Out[•]=}\ \left(R^4-\left(x^2+y^2+z^2\right)^2\right)\left(-6+R^4+2w^4+20x^2+20y^2+20z^2+4w^2\left(-1+x^2+y^2+z^2\right)+\left(x^2+y^2+z^2\right)^2\right)$$

```
In[•]:= localizerHalfAtZero = ReplaceAll[loclzrHalf, z → 0];

In[•]:= gma =
          KroneckerProduct[IdentityMatrix[2], KroneckerProduct[sigmaz, IdentityMatrix[2]]];

In[•]:= MatrixForm[gma];

In[•]:= madeHermitian = ExpandAll[i * localizerHalfAtZero.gma];

In[•]:= MatrixForm[madeHermitian]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
-1+y & 0 & -i & 0 & w-ix & -1 & 0 & 0 \\
0 & 1+y & 0 & -i & -3 & w-ix & 0 & 0 \\
i & 0 & -1-y & 0 & 0 & 0 & -w+ix & -1 \\
0 & i & 0 & 1-y & 0 & 0 & -3 & -w+ix \\
w+ix & -3 & 0 & 0 & 1-y & 0 & -i & 0 \\
-1 & w+ix & 0 & 0 & 0 & -1-y & 0 & -i \\
0 & 0 & -w-ix & -3 & i & 0 & 1+y & 0 \\
0 & 0 & -1 & -w-ix & 0 & i & 0 & -1+y
\end{pmatrix}$$

```
In[•]:= kTheory = FullSimplify[Det[madeHermitian]]
```

$$\text{Out[•]=}\ \left(w^4+2w^2\left(-3+x^2+y^2\right)+\left(3+x^2+y^2\right)^2\right)\left(w^4+2w^2\left(1+x^2+y^2\right)+\left(-1+x^2+y^2\right)\left(15+x^2+y^2\right)\right)$$

```
In[•]:= (1/2) * Total[Sign[Eigenvalues[N[ReplaceAll[madeHermitian, {w → 0, x → 0, y → 0}]]]]]
```

$$\text{Out[•]=}\ -1$$

```
In[•]:= (1/2) *
         Total[Sign[Eigenvalues[N[ReplaceAll[madeHermitian, {w → 0.1, x → 0, y → 0}]]]]]
```

$$\text{Out[•]=}\ -1$$

```
In[•]:= (1/2) * Total[Sign[Eigenvalues[N[ReplaceAll[madeHermitian, {w → 0, x → 2, y → 0}]]]]]
```

$$\text{Out[•]=}\ 0$$

In[•]:=
```
(1/2) * Total[Sign[Eigenvalues[N[ReplaceAll[madeHermitian, {w → 2, x → 0, y → 2}]]]]]
```

Out[•]=
```
0
```