Math 579-001, Spring 2011, TR 11:00-12:15

Relativistic Electrodynamics and Light: Classical Theory

Instructors: Jim Ellison[1] and Klaus Heinemann

We will study the dynamics of N classical relativistic charged particles in external electric and magnetic fields where for $N > 1$ the collective field, by which each particle acts on the other $N-1$ particles, is taken into account. The particle dynamics is governed by the Lorentz force which contains both the collective and external fields. An important application is to modern synchrotron light sources, e.g., storage rings and X-ray free electron lasers [see www.lightsources.org and, e.g., the Advanced Photon Source (APS) and Linac Coherent Light Source (LCLS)].

A mathematical formulation is thus given by the Maxwell equations for the collective fields coupled with the ODE's governing the particle motion. Most of our work will be based on consequences of this Lorentz-Maxwell system for N particles. We are especially interested in the case where N is very large, $O(10^{10})$, and the particles are confined to *small* bunches, so approximations (e.g., statistical mechanical approaches such as the BBGKY or Klimontovich hierarchies) are important. We will consider both non-collective and collective aspects of the Lorentz-Maxwell system as well as the Vlasov-Maxwell mean field approximation. The later is an important approximation where the discrete particle bunch phase space density can be replaced by a smooth density and the collective field by a smooth field; It is a kinetic theory, similar in spirit to magnetohydrodynamics and continuum mechanics.

The first part of the course will emphasize non-collective aspects. We thus begin with a detailed study of the Maxwell equations for an arbitrary source. The special cases of synchrotron, undulator and channeling radiation, for single particles, will be discussed, including calculations of the associated radiated power. Thus Poynting's theorem and special relativity will be introduced as needed. At times the radiation reaction problem (photons and QED were needed to resolve it) will be discussed, including the run-away solutions of the Lorentz-Dirac equation which modifies the Lorentz force. The resultant radiation damping will be considered for example in the context of small electron bunches in storage rings. Much of this is standard in Electrodynamics books (e.g., chapters 6, 11, 12, 14 and 16 in Jackson's *Classical Electrodynamics*, 3rd edition), however, our emphasis will be on the mathematical analysis.

The second part of the course will emphasize collective aspects of the Lorentz-Maxwell system, where collective fields act on the particles. This will include the important Vlasov-Maxwell approximation. The case of arbitrary planar motion will be emphasized including a parallel numerical algorithm which we have successfully implemented; the general case is computationally out of reach at the moment. Finally we discuss 1D and 3D approximations for collective effects in *tiny* electron bunches moving through undulators which is crucial for X-ray free electron lasers. Here the discreteness plays a significant role; the Vlasov-Maxwell approximation is not sufficient. We discuss *microbunching* which is a collective effect causing a coherence which leads to a large increase in radiation power and the light *extraordinaire* of the X-ray free electron laser (e.g. LCLS at the SLAC National Accelerator Laboratory).

Continued on Reverse Side

[1] ellison@math.unm.edu

The course is meant to be self contained and accessible to graduate students in mathematics, physics and engineering. Open problems will be mentioned as we go. Methods of applied mathematics for ODE's and PDE's will be emphasized and will include: Solutions of the inhomogeneous wave equation using the theory of distributions and spherical means. Method of characteristics for first order PDE's. Case-van Kampen normal modes for linearized Vlasov. Linear Hamiltonian systems. ODE stable manifold theory. Stochastic differential equations (SDEs) and associated Fokker-Planck equation. Method of averaging for ODEs and SDEs. Slowly varying wave approximation. Computational approaches to Vlasov-Maxwell system, which requires high performance computing techniques. Density estimation from mathematical statistics. Villani's Fields Medal work[2] on the Landau damping of kinetic theory could be discussed given time and interest.

There is no good text for the course and so class notes, with appropriate references, will be made available. We will be taking material from

1. *Classical Electrodynamics* by J.D. Jackson, 3rd edition
2. *Partial Differential Equations* by L.C. Evans, 2nd edition
3. Several books on distribution theory with applications to hyperbolic (e.g. Maxwell) equations.
4. Material on the Klimontovich-Dupree phase space density and evolution equation from books by Klimontovich and others. This gives the basis of the Lorentz-Maxwell system.
5. *Introduction to Hamiltonian Dynamical Systems and the N-Body problem* by K.R. Meyer, G.R. Hall, D. Offin
6. *Stochastic Methods* by C. Gardiner, 4th edition
7. *Averaging Methods in Nonlinear Dynamical Systems* by J.A. Sanders, F. Verhulst, J. Murdock
8. *Kernel Smoothing* by M.P. Wand, M.C. Jones (Contains material on density estimation)
9. *Introduction to the Physics of Free Electron Lasers* by K.-J. Kim, Z. Huang, R. Lindberg, June 10, 2010 Lecture Notes from US Particle Accelerator School (USPAS)
10. *Ultraviolet and Soft X-Ray Free-Electron Lasers* by P. Schmuser, M. Dohlus, J. Rossbach
11. *The Physics of Free Electron Lasers* by E. L. Saldin, E.V. Schneidmiller, and M.V. Yurkov

For more information please contact J. Ellison at ellison@math.unm.edu. A course web site is being created at `www.math.unm.edu/~ellison`.

[2]`http://www.icm2010.org.in/prize-winners-2010`