Errata/Clarifications

Harmonic Analysis. From Fourier to Wavelets

by María Cristina Pereyra and Lesley Ward

- Chapter 2
  1. Page 33, Figure 2.3, line 5: replace $2 < p_1 < \infty$ by $2 < p_2 < \infty$.
- Chapter 3
  1. Page 56, 2nd displayed formula: sum should over $0 < |n| \leq N$.
  2. Section 3.2.2: replace n by $|n|$ in the following places:
     * page 63 line -7 in last displayed formula in denominator on the right-hand-side replace n^k by $|n|^{-k}$
     * page 63, last line: replace n^{-k} by $|n|^{-k}$.
     *page 64, line 8: replace $(\pi n)^{-1}$ by $(\pi|n|)^{-1}$ and replace n^{-1} by $|n|^{-1}$.
     * page 66, Exercise 3.22 displayed formula: replace $\frac{C}{n}$ by $\frac{C}{|n|}$.
  3. Page 64, Theorem 3.16 and Corollary 3.17: missing hypothesis of continuity (counterexample f equal to zero except at one point, as pointed out by Paul Tupper, Associate Professor, Canada Research Chair, Department of Mathematics, Simon Fraser University). Statements should read as follows,
     * Theorem 3.16. Let $f : \mathbb{T} \to \mathbb{C}$ be continuous, 2π-periodic and integrable,...
     * Corollary 3.17. Assume $f \in C(\mathbb{T})$. If ...
- Chapter 4
  1. Page 88, David: "Exercise 4.16, starts with an induction argument with $k \geq 1$ and $m = 0$. It then says to check by induction when $k \geq 0$ and $m = 0$. For induction, shouldn't this be $k \geq 1$ (start with the base case) and go up? Now you already know that if f and g are continuous then so is $f * g$, but that is not really germane to the problem."

     Perhaps rephrase, we want induction, base case $n = m = 0$ is already known, do the case $k \geq 1$ and $m = 0$ to gain insight in how to go about the inductive step.
  2. Page 88, second displayed formula: missing dy in the second integral.
  3. Page 89, Table 4.1: first item derivative/polynomial in Frequency column must assume $n \neq 0$.
  4. Page 91, Exercise 4.22, 2nd line: remove absolute values in the integrand, that is replace $\int_{-\pi}^{\pi} |K(\theta)|\, d\theta = 2\pi$ by $\int_{-\pi}^{\pi} K(\theta)\, d\theta = 2\pi$.
  5. Page 99, Table 8.1(d)(e): missing 2π in the exponents for both Fourier and inverse Fourier transforms on the left-hand-side column.
  6. Page 100, Exercise 4.39: replace "integrable" by "Riemann integrable".
- Chapter 5

1. Page 108, last line and last displayed formula: remove 2π from the exponent, that is write: $S_N f(\theta) = \sum_{|n|\leq N} \widehat{f}(n)e^{in\theta}$.
2. Page 109, line 6: David Cruz-Uribe suggests to include a short explanation on why the L^2-norm of f is referred as the energy of f...

   "It took me a while (with the help of the PDEs person in the next office) to remember why: if u is the solution of the wave equation on $\mathbb{R}^n$, then $\|\nabla u\|_2$ is constant in time because the integral of the gradient (in both space and time) is the total energy in the system.

   You should probably define this somewhere! It would probably make a good project to have students go off and read about the wave equation and the computation of total energy."
3. Page 111, Exercise 5.8: Notation was confusing for students (some took A^N for the Nth power not the Nth sequence). Are the last two instructions inverted?
4. Page 121, first line: replace "continuous functions g. [..], take $g \in C(\mathbb{T})$" by "twice-continuously differentiable functions g. [..], take $g \in C^2(\mathbb{T})$" .
5. Page 121, Exercise 5.28: * Replace "for continuous functions $g \in C(\mathbb{T})$" by "for twice continuously differentiable functions $g \in C^2(\mathbb{T})$". * in the limit replace $\|S_n g - g\|_{L^p(\mathbb{T})}$ by $\|S_N g - g\|_{L^p(\mathbb{T})}$

- Chapter 6

1. Page 138, Exercise 6.20: Add comment "Note that in matrix language $\langle v, w\rangle = v^t\overline{w}$ when v, w are considered as column vectors".
2. Page 155, Exercise 6.47, in the displayed matrices: in the 8x8 middle permutation matrix in the right-hand-side, the 6th and 7th rows maybe should be permuted (check the calculation).
3. Page 159, Project 6.9(e) last line: replace $\chi : G \to \mathbb{C}$ by $\chi : G \to S_1$ where $S_1 := \{z \in \mathbb{C} : |z| = 1\}$, the unit circle.
4. pages 159-160, Project 6.9 add at the begining: a good reference is [SS03] Section 7.2 and also see Chapter 2 for an application to the distribution of primes (Dirichlet's problem).

- Chapter 7

1. Page 164 line -10: replace "stablish" by "establish".
2. Page 169, Table 7.1: in item (g) Frequency column need $\xi \neq 0$.
3. Page 176, Exercise 7.20: for (j) we need inversion formula. Replace by:

   *Exercise 7.20: Verify property (i) in Table 7.1: if $f, g \in \mathcal{S}(\mathbb{R})$, then $\widehat{f * g}(\xi) = \widehat{f}(\xi)\widehat{g}(\xi)$.

   Add a new exercise after Exercise 7.30.

   New exercise: Verify property (j) in Table 7.1: if $f, g \in \mathcal{S}(\mathbb{R})$, then $\widehat{fg}(\xi) = \widehat{f} * \widehat{g}(\xi)$.

4. Page 179, Exercise 7.25: Replace by:

   *Exercise 7.25: Prove Theorem 7.24. Does Theorem 7.24 remain true if f is only assumed to be continuous and integrable, so that the convolution is well defined? If not, what additional assumptions on f you could add to make it valid?

   Need assumptions of bounded and uniformly continuous, here is a counterexample constructed by David and his student: Let f be defined as zero except on the intervals $[n, n + 2/n^3]$, and on these intervals, let the graph of f be an isosceles triangle of height n. Then

   $$\int f(x)\, dx = \sum_{n=1}^{\infty} n^{-2} < \infty,$$

   so f is continuous and in $L^1(\mathbb{R})$.

   Let $K_t(x) = t^{-1}K(x/t)$, where K is a C^∞ function with $\mathrm{supp}(K) \in (-1, 1)$. Then for all t sufficiently small ($t < 1/n^3$ should work) I get that

   $$K_t * f(n) = n^4 t \int_0^1 K(u)du$$

   and this does not converge to 0 uniformly for all n.

5. Page 183, Exercise 7.32: revise Hint, since these are complex valued one needs to consider $f \pm ig$ also... in fact rewrite as follows:

   * Hint: use Plancherel on $f+g$ to conclude the real parts are equal and on $(f+ig)$ to conclude the imaginary parts are equal.

6. Page 188, Project 7.8(e) in lines -1, -3 and -5, replace $1/(i\pi z)$ by $i/(\pi z)$ (check before doing it!).

- Chapter 8

  1. Page 201, Table 8.2 Item (e) in Frequency column: replace $\widehat{\tilde{T}} = -\tilde{\widehat{T}}$ by $\widehat{\tilde{T}} = \tilde{\widehat{T}}$.
  2. Page 218-219, Project 8.8: Can add sources based on Cameron's Honor's Thesis. She found some very good references includign Tao's trick to prove famous inequalities in the group context... (check her thesis).

- Chapter 9

  1. Page 225, Definition 9.3, line 3: replace $\{g_{n,k}\}_{j,k\in\mathbb{Z}}$ by $\{g_{n,k}\}_{n,k\in\mathbb{Z}}$.
  2. Page 243, Last line Section 9.4.4 replace by: "We conclude that $Q_jf(x) = \sum_{I\in\mathcal{D}_j}\langle f, h_I\rangle h_I(x)$ since for each $x \in \mathbb{R}$ there is exactly one $I \in \mathcal{D}_j$ such that $x \in I$."

- Chapter 10

1. Page 262, Key Property: replace by

   * Key Properties: If $f(x)$ is in V_0 then all integer translates $f(x-k)$ are in V_0. If $f(x)$ is in V_j then $f(2x)$ is in V_{j+1}.
2. Page 286, line -6: add absolute values. Replace $G(\pm 1/2) = 1$ by $|G(\pm 1/2)| = 1$.
3. Page 287, Example 10.38 first and second displayed formulas, almost interchange right-hand-sides, that is $H(\xi) = \chi_{[-1/4,1/4)(\xi)}$ and $G(\xi) = e^{2\pi i\xi} H(\xi + 1/2) = e^{2\pi i\xi}\chi_{[-1/2,-1/4)\cup[1/4,1/2]}(\xi)$.
4. Page 291, last line, Page 292, line 5: replace $\widehat{\varphi}(0) = 1$ by $|\widehat{\varphi}(0)| = 1$.

- Chapter 11

1. Page 317, line 3: replace "holds" by "hold".
2. Page 317, lines 5 and 6 (displayed formulas): replace n in the sum and in the summands by m (for aesthetic reasons, m is used in the previous page).

- Chapter 12

1. Page 336, Section 12.3.1, line 4: replace "$\mathcal{D} = \bigcup_{j\in\mathbb{Z}} \mathcal{D}$, where $I \in \mathcal{D}$" by"$\mathcal{D} = \bigcup_{j\in\mathbb{Z}} \mathcal{D}_j$, where $I \in \mathcal{D}_j$"
2. Page 339, line -7, item (ii) Dilation: in the right-hand-side of the equality should be $\mathrm{Sha}_{\delta_a(r,\beta)}(\delta_a f)$ instead of $\mathrm{Sha}_{\delta_a}(r,\beta)(\delta_a f)$.
3. Page 359, Section 12.7.1. : be careful with the analytic extension of f and the Cauchy formula... We say that F is twice the analytic extension.... so that explains the missing 1/2 in the formula... (check!).

People that have contributed to this list

- David Cruz-Uribe (Trinity College, taught a reading course in Spring 2013), and his student Philip Cho.
- Paul Tupper (Simon Fraser University, teaching in Summer 2014 a class of about 12 students with 2 courses advanced calculus + some applied Fourier in PDE classes).
- Jens Lorenz (UNM, Spring 2015).
- UNM students Math 472/572.

  * Fall 2012: Wang, David Weirich, Cameron Lavigne (also wrote Honor's Thesis defended with honors Dec 2013).

  * Fall 2013: Nuriye Atasaver (Wrote MS thesis on Petermichl's representation of Hilbert transform defended in Dec 2014).