Midterm 2 (Take home) MATH 327 - FALL 2008
Discrete Mathematical Structures
Due on November 25, 2008

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There are seven problems, each worth 10 points, that you should return to me on Tuesday November 25, 2008. There are four bonus problems that you can attempt for extra credit, and can be returned later than the due date.

1. Determine whether the following functions are one-to-one and/or onto. If a given function is a bijection, find its inverse. Pay attention to the prescribed domain and target.
   (a) $g : \mathbb{Q} \to \mathbb{Q}$, where $\mathbb{Q}$ denotes the rational numbers, and
   $$g(r) = \frac{5-2r}{3}, \quad \text{for all } r \in \mathbb{Q}.$$
   (b) $h : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, where $\mathbb{Z} \times \mathbb{Z} = \{(a,b) : a,b \in \mathbb{Z}\}$, and
   $$f(a,b) = ab, \quad \text{for all } (a,b) \in \mathbb{Z} \times \mathbb{Z}.$$
2. We are given functions $f : A \to B$ and $g : B \to C$. Suppose that $g \circ f$ is one-to-one and f is onto, show that g is one-to-one.
3. Illustrate the Euclidean algorithm by showing that 571 and 386 are relatively prime. Find integers m and n such that $571m + 386n = 10$.
4. Suppose a, b,and c are integers such that c divides ab. If c and a are relatively prime, show that c must divide b.
5. Use mathematical induction to show that $f(n) = 10^{n+2} + 10^n + 1$ is divisible by 3 for all $n \geq 1$.
6. Prove by mathematical induction that if the cardinality of a set A is n then the cardinality of its power set $\mathcal{P}(\mathcal{A})$ is 2^n.
7. Let the sequence $\{a_n\}_{n\geq 0}$ be defined recursively by $a_0 = 0$, $a_1 = \frac{1}{3}$, and for all $n \geq 2$, $a_n = \frac{1}{2}(a_{n-1} + a_{n-2})$. Show using mathematical induction that $a_n = \frac{2}{9}\left(1 - \frac{(-1)^n}{2^n}\right)$ for all integers $n \geq 0$.
8. (Bonus) Exercise 16 in page 97.
9. (Bonus) Exercises 15 in page 167.
10. (Bonus) Exercise 56 in page 170.
11. (Bonus) Exercises 23-24 in page 175.