MIDTERM # 1 - MATH 327 - Fall 2008
October 2, 2008

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Read carefully all the problems. There are 6 problems in the exam. Choose 5 problems to do for 20 points each. Indicate the problems you have chosen. The remaining problem will be 20 bonus points. Good luck!!!!

1. Write down the negation of each of the following statements. Only the original statement or its negation is true, not both. Which one? Explain. Remember that an integer n is a multiple of an integer k if k divides n.

   (a) The number 15 is a multiple of 2 and a multiple of 3.

   (b) The number 15 is a multiple of 2 or a multiple of 3.

   (c) There is an integer that is a multiple of 2 and a multiple of 3.

   (d) All integers are multiples of 2 or multiples of 3.

2. Let p and q be two statements. Remember that $\neg p$ denotes the negation of the statement p.

   (a) Show that the implication $p \to q$ is logically equivalent to the implication $\neg q \to \neg p$. The second implication has a special name, what is it?

   (b) Show that the following compounded statement is a contradiction,

$$(p \to q) \quad \text{and} \quad (p \text{ and } \neg q)$$

3. Show that the following pairs of sets are not equal by exhibiting an element of one that is not an element of the other. For each pair decide whether one set is a subset of the other or not, in the first case indicate which set is the subset. (Recall that $\mathcal{P}(S)$ is the power set S.)

   (a) $A = \{1\}, \quad B = \{1, \{2\}\}$.

   (b) $C = \mathcal{P}(\{1, 2\}), \quad D = \{\{1\}, \{2\}, \{1, 2\}\}$.

   (c) $E = \{1, 2\} \times \{2, 3\}, \quad F = \{2, 3\} \times \{1, 2\}$.

4. In this problem you are asked to write complete proofs. Please indicate whether you are using a direct argument, and argument by contradiction, or a contrapositive argument.

   (a) Show that if $(n^2 + 6n)$ is even then n is an even integer.

   (b) Show that $\sqrt{3}$ is not a rational number.

   (c) Show that if $A \subset B$ then $A \cap B = A$.

5. Consider the set $A = \{1, 2, 3, 4\}$.

   (a) Verify whether the given relation is reflexive, symmetric, antisymmetric, or transitive:

   $$R = \{(1,1), (2,1), (2,2), (2,3), (3,1), (3,3), (4,4)\}$$

   (b) Verify whether the given relation is reflexive, symmetric, antisymmetric, or transitive:

$\sim$	1	2	3	4
1	*			
2			*	
3		*	*	
4				*

   (c) Find an equivalence relation on A (construct the table) such that the equivalence classes are the sets $\{1\}$, $\{2, 4\}$, and $\{3\}$.

   (d) Can the sets $\{1\}$, $\{2, 4\}$, and $\{1, 3\}$ be the equivalence classes of an equivalence relation on A?

6. Determine, with reasons, whether or not each of the following defines an equivalence relation on the given set A. If it does, can you describe the equivalence classes?

   (a) Let A be the set of all straight lines in the plane. Given two lines ℓ, m in the plane, $\ell \sim m$ if and only if ℓ is perpendicular to m.

   (b) Let A be the set of all circles in the plane. Given two circles a and b in the plane, $a \sim b$ if and only if a and b have the same radius.