Review 1 - MATH 313:COMPLEX ANALYSIS - Fall 2013

1. Find each of the following limits if they exist otherwise justify why they don't exist.[1]

   (a) $\displaystyle\lim_{z\to 2+i} |z^2-4|$.   (b) $\displaystyle\lim_{z\to\infty} \frac{3z^2+iz}{z^2+2z+3i}$.

   (c) $\displaystyle\lim_{z\to\infty} \sin z$.   (d) $\displaystyle\lim_{z\to 3i} \frac{e^z}{z^2+9}$.

2. Given the following functions:

   (i) $\displaystyle\frac{(z+2)^3}{(z^2+2iz-1)^4}$,   (ii) $z|z|$,   (iii) $\text{Im}z - i\text{Re}z$,   (iv) $3\bar{z}$.

   (a) Is any of these functions continuous everywhere? Specify.
   (b) Is any of these functions nowhere differentiable? Specify.
   (c) Is any of these functions analytic somewhere? Specify the domain of analyticity for each one that qualifies.
   (d) Is any of these functions an entire function? Specify.

3. Given the following functions:

   (i) $\displaystyle f(z) = \frac{z^2-|z|^2}{2}$,   (ii) $g(z) = (\bar{z})^2 + 4z\text{Re}z - 4(\text{Re}z)^2$.

   (a) Is any of these functions entire? Specify.
   (b) Is the imaginary part of any of these functions a harmonic function on $\mathbb{C}$?

4. Verify whether the following equalities hold:

   (a) $\text{Log}\big((1+i)^3\big) = 3\text{Log}(1+i)$,   (b) $\text{Log}\big((-1+i)^2\big) = 2\text{Log}(-1+i)$.

5. Write each of the following numbers in the form $a+ib$, where $a, b \in \mathbb{R}$.

   (a) $\displaystyle\left(\frac{i}{2+2i}\right)^{1/5}$,   (b) $|\cos(z)|$ for $z = 3-i$,   (c) z^i for $z = 2-2i$.

6. Find a function analytic on the open disc $|z|<1$ whose real part is $3xy^2 - x^3$. Can you find one that is not analytic?

7. Find a function $\phi(x,y)$ that is harmonic in the wedge $r>0$, $\frac{\pi}{2} < \theta < \frac{5\pi}{4}$, and takes the values $\phi = 3$ on the vertical side and $\phi = -1$ on the lower side of the wedge.

8. Does $\text{Im}(e^{iz}) = \sin z$ hold for every $z \in \mathbb{C}$?

9. Find the derivative of $f(z) = \sinh(z^2) - 2z\cos(3z)$. Is this an analytic function?

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[1] Note the following definitions: 1) $\lim_{z\to\infty} f(z) = L$ if and only if $\lim_{z\to 0} f(1/z) = L$, 2) $\lim_{z\to z_0} f(z) = \infty$ if and only if $\lim_{z\to z_0} \frac{1}{f(z)} = 0$, 3) $\lim_{z\to\infty} f(z) = \infty$ if and only if $\lim_{z\to 0} \frac{1}{f(1/z)} = 0$.