MIDTERM # 1 - MATH 180 - Sections 009/013 - FALL 2008
Calculus I for Biology and BA/MD
September 30, 2008

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There are 6 problems, each worth 20 points. No books or notes allowed. Calculators can be used, but a correct answer to a problem without an explanation on how you got the answer will receive minimal credit. Good luck!

1. A culture of bacteria is known to have total volume $15 \times 10^{-3}\text{m}^3$ (cubic meters), the density of bacteria per unit of volume is 4×10^3 bacteria per cm^3 (number of bacteria per cubic centimeter). What is the total number of bacteria that the culture contains? Remember that $1\text{m} = 10^2\text{cm}$.

2. Consider the functions $f(x) = e^{3x+1}$ and $g(x) = 2x - 1$. Find the composition $f \circ g\,(x)$, and its inverse function $(f \circ g)^{-1}(x)$. Check that the functions obtained are inverses of each other.

3. Consider the data in the following table describing the number of wasps that can develop inside caterpillars of different weights.

Weight of Caterpillar (g)	Number of Wasps
0.5	80
1.0	115
1.5	150
2.0	175

   (a) Graph the data. Which point does not lie on the line?

   (b) Find the equation of the line connecting the first two data points.

   (c) How many wasps does the function predict would develop a caterpillar weighting 0.7 g?

4. Cell volume follows the discrete-time dynamical system $\nu_{t+1} = 1.5\nu_t$, where t is measured in hours, and with initial volume $1350\mu\text{m}^3$.

   (a) When will the volume reach $3250\mu\text{m}^3$?

   (b) What will the volume be after 24 hours?

5. A population of bacteria doubles every hour, but 1.0×10^6 individuals are removed after reproduction to be converted into valuable biological by-products. The population begins with $b_0 = 3.0 \times 10^6$ bacteria.

   (a) Find the population after 1, 2 and 3 hours.

   (b) Write the dynamical system, and find equilibria points (if they exist).

6. A discrete-time dynamical system has updating function $f(b_t) = b_{t+1}$, where

$$f(x) = \begin{cases} \frac{2}{3}x & \text{if } x > 1.5 \\ \frac{2}{3}x + 1 & \text{if } x \leq 1.5 \end{cases}$$

   (a) Draw the graph of the updating function and cobweb starting from $b_0 = 2$. Describe what happens to b_t as t gets larger. Are there equilibria points?

   (b) Find an initial value b_0 such that $b_2 = b_0$. For that initial value, what happens to b_t as t gets larger?