Final Exam - MATH 180 - Sections 009/013 - FALL 2008

Calculus I for Biology and BA/MD

December 15, 2008

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There are 10 problems, for a total of 100 points. No books or notes allowed. Calculators can be used, but a correct answer to a problem without an explanation on how you got the answer will receive minimal credit. Good luck!

1. A culture of bacteria is known to have total volume 0.125m^3 (cubic meters), the density of bacteria per unit of volume is 8×10^3 bacteria per cm^3 (number of bacteria per cubic centimeter).

   (a) What is the total number of bacteria that the culture contains? (Remember that 1m= 10^2cm.)

   (b) It is observed that the volume of the culture of bacteria decays as a function of time (time t in hours) according to the following exponential function

   $$V(t) = e^{-3t}V(0).$$

   If the volume at time $t = 0$ is 0.125m^3 (cubic meters), what will the volume be after 5 hours?

   (c) How many hours do we have to wait to see a third of the initial volume? Does your answer depend on the initial volume?

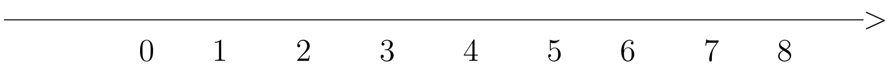
2. A bacterial population doubles every hour, but 10^5 individuals are removed after reproduction. The population begins with $b_0 = 4 \times 10^5$ bacteria.

   (a) Write the dynamical system, and find the population after 1, 2 and 3 hours.

   (b) Find the updating function, the equilibrium points (if they exist), and classify them.

   (c) Consider a bacterial population that doubles every hour, but h individuals are removed after reproduction. Find the equilibrium point, does it make sense?

3. A discrete-time dynamical system $b_{t+1} = f(b_t)$, has updating function f, whose graph is given,



(a) Cobweb starting from $b_0 = 1$. Describe what happens to b_t as t gets larger.

(b) Identify and classify all equilibrium points (stable, unstable, or neither).

4. Determine whether the following limits exist or not. If they do, find the limit.

   (a) $\lim_{t \to 1} \left(\ln(3t) + t^3 \right)$

   (b) $\lim_{t \to 0} \frac{e^t - 1}{t}$

   (c) $\lim_{z \to 2^+} \frac{3}{z - 2}$

   (d) Let $f(x) = \begin{cases} 2x + 1 & \text{if } x > 0 \\ -1 & \text{if } x \leq 0. \end{cases}$

   Find right and left limits of $f(x)$ as x approaches 0.

   Does $\lim_{x \to 0} f(x)$ exist?

5. Given the function $f(t) = 3t^3 - 9t^2 + 5$

   (a) Find the average rate of change of f between $t = 1$ and $t = 3$.

   (b) Find the equation of the secant line to the graph of f going through the points $(1, f(1))$ and $(3, f(3))$.

   (c) Find the first and second derivatives of f, find critical points (points where $f'(t) = 0$ and $f''(t) = 0$).

   (d) Find the equation of the tangent lines to the graph of f at $t = 0$ and at $t = 2$.

   (e) Sketch the graph of f for $-1 \leq t \leq 3$ with the information so far gathered. Plot on the graph the secant and tangent lines found in parts (b) and (d).

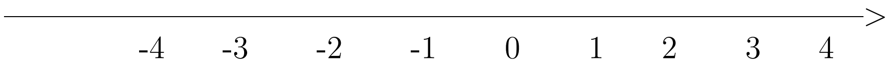
6. Find the derivatives of the following functions

   (a) $g(t) = t^2 e^t$.

   (b) $f(x) = \dfrac{2x+3}{1-4x}$.

   (c) $h(y) = \ln(1+y^2)$.

7. The graph of a function $f(x)$ for $-4 \leq x \leq 4$ is given by:



(a) Is the function continuous? Indicate points of discontinuity.

(b) Indicate points where the function is clearly not differentiable, and points where the derivative is zero.

(c) Find intervals where the function f is increasing/decreasing.

(d) Find intervals where the function f is concave up/down, and inflection points.

(e) Sketch the graph of the derivative for $-3 \leq x \leq 3$.

8. Determine the following antiderivatives/indefinite integrals,

   (a) $\int x \ln x \, dx.$

   (b) $\int 2x\, e^{-x^2} \, dx.$

   (c) Find a function $F(x)$ such that its derivative

$$F'(x) = e^{x+1} + 2, \qquad \text{and} \quad F(-1) = 1.$$

9. This exercise evaluates your knowledge of the geometric meaning of the definite/improper integrals.

   (a) Find the area under the graph of the function $f(x) = \sin x$, above the X-axis and for $0 \leq x \leq \pi$

   (b) Calculate the improper integral, $\int_1^{\infty} \frac{1}{x^3}\, dx$, explain the geometric meaning of the number obtained.

   (c) Given the graph of the function g, find the value of $\int_1^{7} g(y)\, dy$,

10. The following table records the velocity (miles/hours) read in a vehicle's odometer every half an hour.

Time t (hours)	Velocity at t (miles/hour)	Distance traveled during half-hour interval	Distance traveled at time t (miles)
0.5	20		
1.0	34		
1.5	40		
2.0	56		
2.5	38		
3.0	16		

(a) Estimate the distance traveled after 3 hours.

(b) If the initial position was 0, what would be a reasonable curve describing the position at time t for $0 \leq t \leq 3$? It might be useful to fill in the fourth column in the table.