

Math 569: Selected Topics in Analysis, Fall 2011 Oscillatory Integrals in Analysis and PDE

Instructor: Matthew Blair
Meets MWF 12-12:50pm in SMLC 352

As the name suggests, oscillatory integrals arise when one attempts to integrate a function which possesses some oscillatory behavior (usually characterized by trigonometric or exponential functions). The cancellation introduced by such behavior means that these integrals are often better behaved than one might expect just by integrating the absolute value of the function. The Fourier transform of a function is an important example of an such an integral. As a consequence, oscillatory integrals have deep connections with problems in Fourier analysis, but also partial differential equations and special functions.

At the core of this course will be a rigorous introduction to the Fourier transform and the method of stationary phase, two topics which have wide ranging applications in pure and applied mathematics. We will then examine some of the implications of these topics in PDE, particularly what they tell us about the decay of solutions to wave and Schroedinger equations. Finally, we will consider certain linear operators arising in the study of the Fourier transform which are defined by oscillatory integrals and study their mapping properties.

Course outline:

- A crash course in measure theory and L^p spaces
- A thorough introduction to the Fourier transform
- Method of stationary phase: inequalities and asymptotics
- Applications of stationary phase
 - Asymptotics of Bessel functions
 - Fourier transforms of surface carried measures
 - Elementary lattice point counting problems
- Introduction to Littlewood-Paley theory and elementary singular integrals
- Strichartz's inequalities for the wave and Schroedinger equations
- Oscillatory integral operators
 - Restriction theorems
 - Convergence of Bochner-Riesz means

Prerequisites: Real analysis at the level of 510-511 and elementary complex analysis. A background in functional analysis and measure theory will be helpful, though every effort will be made to see that this is not essential.

Textbook: There is no textbook for the course.