Math 311, Fall 2016
Assignment 6, due Wednesday, October 12

Hand in all of the following problems:

- 3.4: 17, 19, 20, 23, 24, 26, 28
- 5.4: 16, 20, 22, 26
- 5.5: 8, 10
- In §3.4 #28(a), you saw that the Laplacian operator could be interpreted as $\nabla \cdot \nabla f$, in other words, $\nabla^2 f = \text{div}\,(\nabla f)$. Use this and Theorems 4.5, 4.6 in §3.4 of the text to give a second derivation of the formulas for the Laplacian in cylindrical and spherical coordinates in $\mathbb{R}^3$, which you originally looked at in exercises #32 and #33 of §2.5:

$$\nabla^2 f = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$
$$\nabla^2 f = \frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{1}{\rho^2 \sin^2\varphi}\frac{\partial^2 f}{\partial \theta^2} + \frac{2}{\rho}\frac{\partial f}{\partial \rho} + \frac{\cot\varphi}{\rho^2}\frac{\partial f}{\partial \varphi}$$

Notes:

- For exercises #17, #19, and #20 in §3.4, solve the problems in two ways: by using the usual Cartesian coordinates **and** by switching to spherical coordinates, using that $r = \rho$ and $\mathbf{r} = \rho \mathbf{e}_\rho$. The computation in Cartesian coordinates in #20 may be easier if you use the formula in exercise #24.
- When proving formula (5) in Theorem 4.5 of §3.4, start by deriving the following formula (show it, don't take it for granted)

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (\cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}) & (\sin\theta \frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial \theta}) & \frac{\partial}{\partial z} \\ (F_r\cos\theta - F_\theta \sin\theta) & (F_r \sin\theta + F_\theta\cos\theta) & F_z \end{vmatrix}$$

Reading: Review 5.1-5.4 as needed, 5.5, 5.6

Problems to do on your own:

- 3.4: 21, 22, 25, 27, 29
- Miscellaneous exercises in Chapter 3, p. 243: 43, 44
- 5.4: 11, 13, 15, 17, 19, 25, 27
- 5.5: 1, 3, 5, 9, 11
- Review exercises from 5.1-5.4 as needed