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DECAYING TWO-DIMENSIONAL TURBULENCE IN BOUNDED FLOWS

A.H. Nielsen[1], D.J. Torres[2], and E.A. Coutsias[3]

[1]*Association EURATOM-Risø National Laboratory, OFD-128 Risø, DK-4000 Roskilde*
[2]*Physics Dep. and Geophysical Research Center, New Mexico Tech, Socorro, NM 87801, USA*
[3]*Mathematics and Statistics, University of New Mexico, Albuerqueque, NM 87131, USA*

1. Introduction

Two-dimensional turbulence in unbounded or double periodic domains, in Navier-Stokes fluids and plasmas has been investigated intensively during the last decades by means of numerical simulations. The presence of an inertial range in the energy spectrum of these flows has been well documented. As two-dimensional flows exhibit an inverse cascade in the energy, coherent structures comparable with the size of domain will eventually emerge. Thus, the presence of boundaries and the conditions imposed on them will play a significant role in the evolution of the turbulence and the coherent structures, see e.g. [1].

In this contribution we numerically investigate decaying low frequency electrostatic turbulence in a disk geometry. The model equations are identical to the Navier-Stokes equations and are solved using a pseudo spectral method based on a Chebyshev-Fourier expansion of the solutions. We investigate two different kinds of boundary conditions: first, a noslip condition where, due to the strong boundary layers, small scale structures are injected into the interior of the flow, whenever coherent structures interact with the wall. These small scale structures will thus feed the turbulence. Second, the condition of free-slip boundaries with no vorticity generation at the wall. For these two boundary conditions we investigate the time evolution of different quantities such as: energy, enstrophy and angular momentum.

2. Numerical model

We assume a strongly magnetized plasma confined in a cylinder of radius $R = 1$ and the fluctuations are considered to be aligned strictly to the magnetic field. Hence, the analysis is carried out in a plane perpendicular to $\vec{B}_0 = B_0\hat{z}$. We also assume low frequency fluctuation, well below the ion cyclotron frequency. The velocity is, thus, to a first approximation, given by in incompressible $\vec{E} \times \vec{B}_0$ velocity. Furthermore, both electrons and ions are assumed to have zero Larmor radius, and the two continuity equations can be combined into one for the charge density. In dimensionless units the guiding center equations are

$$\frac{\partial \omega}{\partial t} + J(\omega, \psi) = \nu \nabla^2 \omega, \tag{1}$$

where ν is the kinematic viscosity, ω is the charge density (corresponding to the vorticity) in the z-direction and ψ, the electrical potential (corresponding to the stream function), is related to the vorticity by the Poisson equation

$$\nabla^2\psi = -\omega. \tag{2}$$

The Jacobian, in polar coordinates, is given by

$$J(\omega,\psi) = \frac{1}{r}\left[\frac{\partial\omega}{\partial r}\frac{\partial\psi}{\partial\theta} - \frac{\partial\psi}{\partial r}\frac{\partial\omega}{\partial\theta}\right], \tag{3}$$

and is computed at the origin using Cartesian coordinates,

$$J(x,y) = \left[\frac{\partial\omega}{\partial x}\frac{\partial\psi}{\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}\right]. \tag{4}$$

We have used two different boundary conditions, noslip:

$$\left.\frac{\partial\omega}{\partial r}\right|_{r=R} = 0 \quad , \quad \left.\frac{\partial\psi}{\partial\theta}\right|_{r=R} = 0, \tag{5}$$

where vorticity layers are formed in order to satisfy the noslip constrain or free-slip:

$$\left.\frac{\partial\psi}{\partial r}\right|_{r=R} = 0 \quad , \quad \left.\frac{\partial\psi}{\partial\theta}\right|_{r=R} = 0, \tag{6}$$

where no vorticity is produced at the wall.

Numerically we solve Eqs. (1-3) by expanding the solution into series of Fourier modes and Chebyshev polynomials according to:

$$\omega(r,\theta) = \sum_{n=-N/2}^{N/2-1} \sum_{\substack{m=0 \\ m+n \text{ even}}}^{M} \omega_{mn} T_m(r)\exp(in\theta), \tag{7}$$

where $T_m(r)$ represents the Chebyshev polynomial of order m, $r \in [-1,1]$. For further details on the numerical code, see [2].

3. Results

As initial condition we used random noise with a specific length scale. From a double periodic squared box of length 1 we initialize a vorticity field in Fourier space of the form:

$$\omega^*(k_x,k_y) = \exp\left(-\frac{\left(\sqrt{k_x^2+k_y^2}-k_0\right)}{\Delta^2}\right)^2 \exp\left(in\Theta\right), \tag{8}$$

where k_0 is a length scale, Δ is the width in mode space of the noise and Θ is a random phase. This field is then spectrally interpolated onto the grid points of the disk, discarding points laying outside the disk. For such flows we define a Reynolds number as: $Re = u^*R/\nu$, where u^* is the rms value of the velocity field.

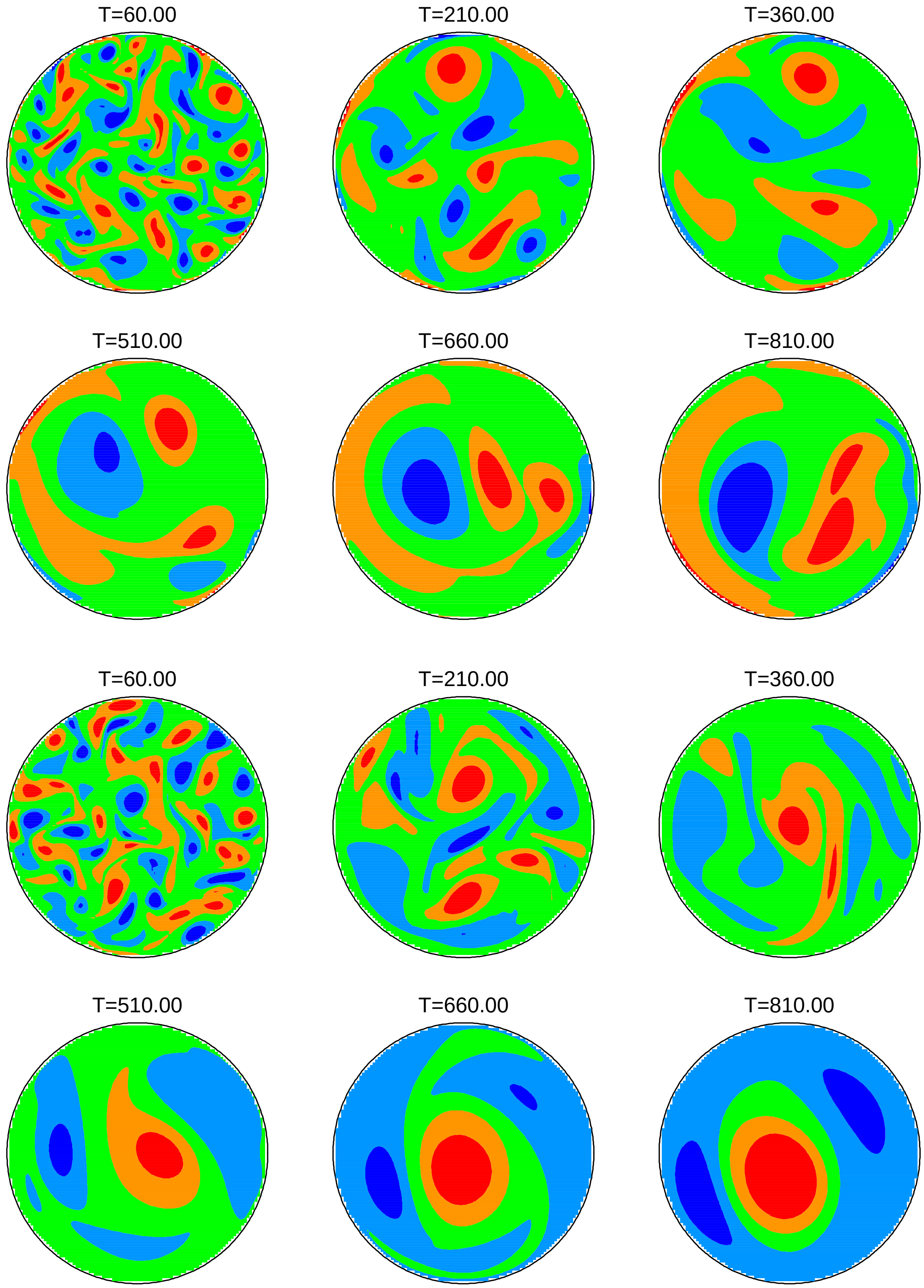

Figure 1. Time evolution of a flow initialized with random initial conditions. Top figure is for noslip boundary conditions and bottom for free-slip. Reddish colors correspond to positive values, bluish to negative. Limits adjusted to the individual pictures. Parameters: $M = 1024$, $N = 512$, $\nu = 2.010^{-5}$, $k_0 = 20$,$\Delta = 10$ and $Re = 1538$.

The time evolution of a flow using Eq. (8) as initial condition is shown in Figure 1 for both noslip and free-slip boundary conditions. The Reynolds number is $Re = 1537$ in both cases. A clear self-organization is observed as a result of vortex merging. At the end of the simulation a dipole with strong boundary layers is observed for the noslip boundary condition whereas a tripole is the end result for free-slip boundary conditions. These results are qualitatively in agreement with other numerical results, see e.g. [1] and, for the noslip case, experimental results in a fluid, see e.g. [3].

The corresponding temporal evolution of enstrophy, $\Omega(t) \equiv \int_D \omega^2 dA$, energy, $E(t) \equiv 1/2 \int_D \vec{u}^2 dA$ and angular momentum, $L \equiv \int_D \vec{u} \times \vec{r} \cdot \hat{e}_z dA$ is shown in Figure 2. The enstrophy and energy has been normalized with their initial values while the angular momentum has been normalized with: $-\sqrt{8E(t)\pi/9}$, .i.e., the angular momentum if the energy at a given time is resulting in a solid body rotation of the fluid. Enstrophy and energy is observed to decay quite rapidly. Even though there are significant differences between the flow fields, see Figure 1, enstrophy has a decay rate of $t^{-1.75}$ and energy have a decay rate of $t^{-0.75}$. The angular momentum is in both cases nearly a linear growing (decaying) quantity, but at the end of the simulation it is still far from solid body rotation, $L = 1$. Note that the sign of the angular momentum is unimportant, a strong positive structure is just as likely to emerge as a negative one.

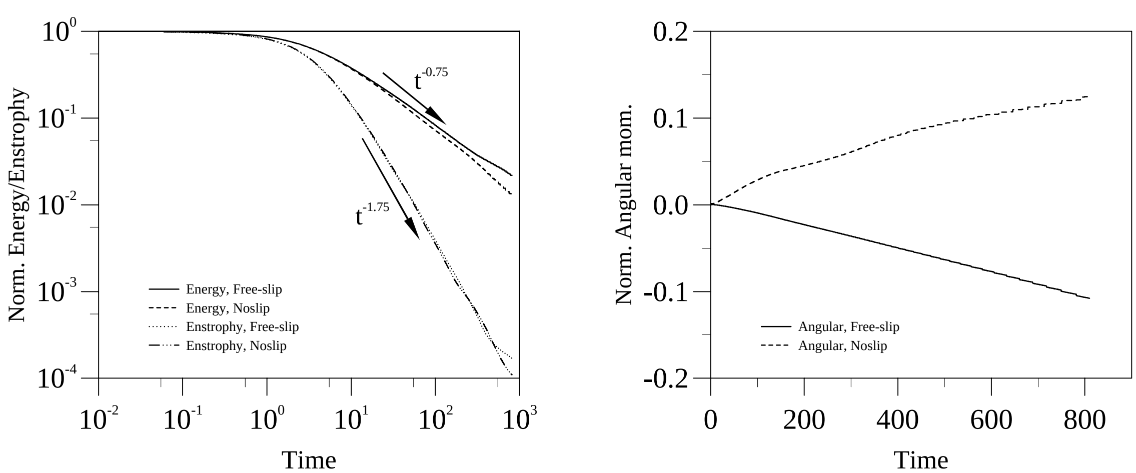

Figure 2. Left: Normalized energy and enstrophy evolution for the runs in Figure 1. Right: Angular velocity normalized with the energy.

References

[1] S. Li, D. Montgomery, and W.B. Jones: JFM **56**(3), 615-639 (1996)

[2] D.J. Torres and E.A. Coutsias: *submitted to* SIAM Journal of Scientific Computing.

[3] H.J.H. Clercx, S.R. Maassen, and G.J.F. van Heijst: PRL **80**(23), 5129 (1998)