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COHERENT VORTICAL STRUCTURES IN TWO-DIMENSIONAL PLASMA TURBULENCE

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ABSTRACT

A laboratory experiment was carried out in order to study the nonlinear saturated stage of the cross-field electrostatic Kelvin-Helmholtz instability in a strongly magnetized plasma. The presence of large vortex-like structures in a background of wide-band turbulent fluctuations was demonstrated by a conditional sampling technique. The formation of the structures was interpreted in terms of an inverse cascade, i.e. as the combined result of small scale structures generated by the instability and a subsequent coalescence of small vortices into larger ones. This interpretation was supported by numerical simulations. The importance of the large scale structures for the turbulent plasma transport across magnetic field lines was analyzed in detail.

INTRODUCTION

Two basically different forms of turbulent transport can be envisaged. In one extreme limit the displacement of a particle can be considered as the result of very many small individual random movements, a limit resembling Brownian diffusion. In the opposite extreme, particles are convected by random bursts occuring at irregular intervals seperated by more or less quiescent periods. Finally, of course a mixture of the two types of processes can be realized. While the limit of Brownian diffusion is well covered in the litterature, the mechanisms for random bursts and their consequences for particle transport are less understood. In the present study we describe an experimental investigation of this problem, carried out in a strongly magnetized plasma. The purpose of the study is twofold. First, methods are developed for detecting and identifying coherent structures, which can cause sporadic bursts of plasma across magnetic field lines. The origin of these structures (in the present case having the form of two-dimensional vortices) is discussed on the basis of a numerical simulation. Next, the statistical importance of these events for the turbulent transport is considered.

EXPERIMENTAL RESULTS

The experimental investigations were performed in the Risø Q-machine. The fluctuations were generated by the Kelvin-Helmholtz instability due to a strongly sheared azimuthal flow in the residual plasma (the scrape-off layer) surrounding the main plasma column (Kent *et al.* 1969, Huld *et al.* 1991). The radially varying dc-electric field $\mathbf{E}_0(r)$ was controlled by the bias on a limiting aperture inserted perpendicularly to the plasma column near the plasma source (i.e. the hot plate). The fluctuations in the floating potential and density were measured by small

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1 mm diam. platinum probes. Fluctuating electric field components were measured by taking the potential difference between two closely spaced probes. The fluctuations were characterized by a broad frequency spectrum for a weak radial electric field $\mathbf{E}_0$, whereas narrow peaks appeared for strong, localized radial electric fields, see Huld *et al.* (1988, 1991). Also the radial component of the turbulent plasma flux was studied. An important observation was that the actual value of this flux was strongly dependent on the shape of the turbulent spectrum. In particular, it was demonstrated (Huld *et al.* 1988) that the internal phase relations between density and electric field fluctuations were important. This phase relation was shown to vary with frequency within the spectrum. A narrow frequency spectrum could have a large amplitude without contributing significantly to the overall average plasma flux.

The presence of relatively long-lived coherent vortical structures, or eddies, was demonstrated by employing a conditional averaging technique (Huld *et al.* 1991), where a signal is analyzed only in time intervals, where a certain prescribed condition is fulfilled in a reference signal. The basic idea of this technique, which can reveal the averaged properties of large structures appearing in the turbulence, is described by Johnsen *et al.* (1987) and Kofoed-Hansen *et al.* (1989). For the present investigations we recorded simultaneously two time series of 200 ms duration obtained with a sampling rate of 125 kHz and an 8-bit resolution. One of the series, the reference, came from the azimuthal electric field component, $\tilde{E}$, the other from the potential fluctuations, $\tilde{\phi}$, measured by a movable probe. Such pair of time series were obtained for positions in a grid of 9×13 points with a 3 mm spatial resolution in the plane perpendicular to the magnetic field. The conditional averaging was subsequently performed numerically for each position of the movable probe. The space-time evolution of the conditionally averaged potential is shown in Fig. 1 for typical plasma parameters with a relatively strong radial electric field. The formation, propagation and beginning decay of a large monopole-like structure is clearly noticable. The propagation velocity is approximately 250 m/s, which is compatible with the averaged plasma $\mathbf{E}_0 \times \mathbf{B}_0/B_0^2$-velocity in the scrape-off layer. Depending on the parameters the dominant structures can appear as monopole or dipole vortices. Monopoles with negative potential, i.e. having a rotation in the same direction as the bulk plasma rotation in the scrape-off layer, tend to dominate in the cases with strong radial electric fields.

Vortices like those described here are not associated with wave-like motion. Rather, fluid or plasma elements are trapped in closed orbits within the structures. Propagating vortices are thus expected to be particularly effective in mediating particle transport by a burst-like process in contrast to slow Brownian motion.

The properties (such as growth or damping) of the averaged vortical structures obtained by the conditional sampling method need not reflect the properties of the vortices in individual realizations (Johnsen *et al.* 1987, Pécseli and Trulsen 1989, Kofoed-Hansen *et al.* 1989 and Nielsen *et al.* 1992). Thus, an apparent damping of the averaged structures may simply be caused by a "smearing-out" due to a spread in the velocities of the individual structures. This question was resolved by performing a coincidence counting of the occurrence of local extrema in the fluctuating potential signals at two points in space separated by a distance, L, in the direction of propagation, as discussed by Nielsen *et al.* (1992). In Fig. 2 we show the distribution of time delays Δt between local minima occurring at two probes (with L=14.6 mm), subject to the condition that the minimum at the first probe is smaller than $\phi_* = -\sigma, -1.5\sigma$ or -2σ , respectively. Here σ denotes the rms-value of the fluctuations. Subsequently the signal from the other probe is searched in a suitably chosen time interval for the deepest minimum, which defines Δt. The distributions obtained for the three values of ϕ_* are essentially identical giving an averaged value of the velocity corresponding to 300 m/s. From results as those shown in Fig. 2 we have derived velocity distributions, and we observed that the potential wells, corresponding to the vortical structures, propagate with a moderate spread in velocity (for the case in Fig. 2 we find $\Delta V \approx 30$ m/s). Thus, it may be argued that the evolution of the conditionally averaged structures of negative potential reflect the properties of individual structures, and is not just a consequence of a spread in their velocities. For these plasma parameters, where monopolar vortices with negative potential were found to

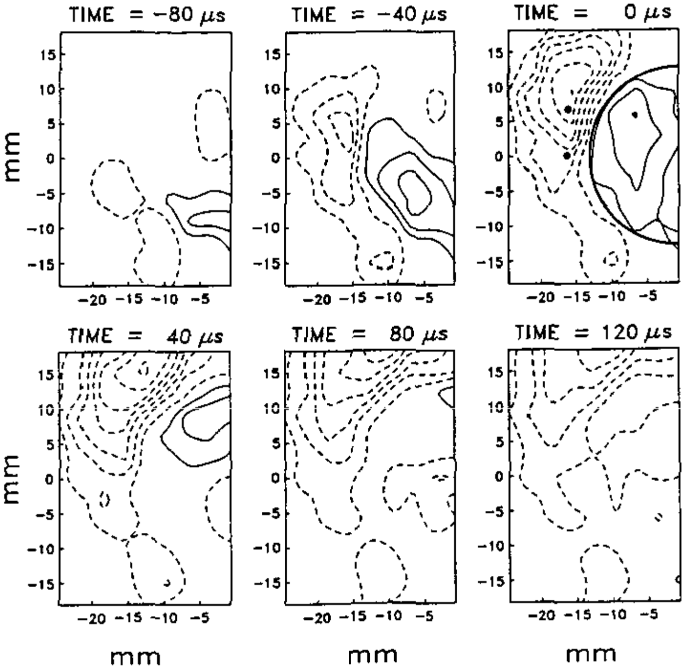

Figure 1: Space-time evolution of the conditional averaged potential. The imposed condition was that $\tilde{E}$ should exceed the value $E_* = 1.5\sigma$ and that $\partial_t \tilde{E} > 0$. Positive potentials are given with full lines, negative ones with dashes lines. The contour inteval is 0.025 V. The heavy semi-circle in the panel for $t = 0\,\mu$s indicates the projection of the aperture opening. Plasma conditions were $n_0(r = 0) = 2 \cdot 10^{15}\,\text{m}^{-3}$ and $B_0 = 0.35$ T.

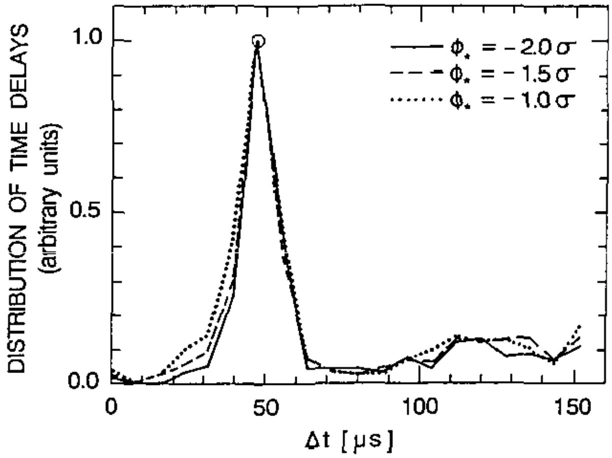

Figure 2: Distribution of time delays Δt between occurence of local potential minima at two probes seperated azimuthally by L = 14 mm. The curves are normalized by the peak value, indicated by a circle. Plasma conditions were $n_0(r = 0) = 7 \cdot 10^{14}\,\text{m}^{-3}$, $B_0 = 0.285$ T.

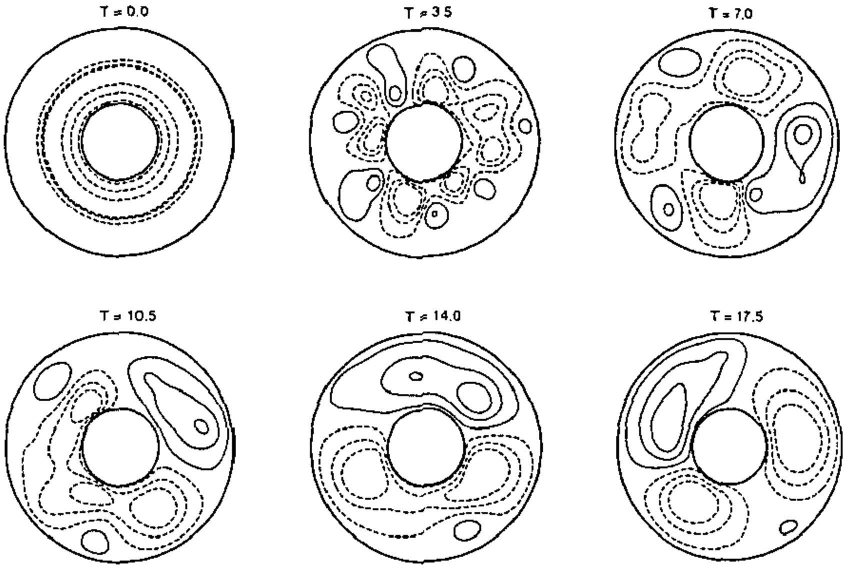

Figure 3: Space-time evolution of the perturbed potential, ϕ, for an initially unstable potential profile. The equilibrium or vacuum profile $\phi_0 \sim ln r$ has been subtracted. Positive ϕ-values are shown with full lines, negative ϕ-values with dashed lines. For this case $\nu = 0.001$.

dominate, a similar analysis of the occurrences of local maxima gave a different result. Although a peak in the distribution of time delays could be discriminated, it was not significantly above the background level, and it was not possible to assign a velocity distribution for potential humps.

The formation of the large scale vortical structures is believed to be accomplished by an inverse spectral cascade, which is characteristic for two-dimensional turbulence (Seyler *et al.* 1975) and was studied previously for our experimental conditions (Huld *et al.* 1988). The saturated turbulent spectrum will thus be dominated by the large scale structures, and it contain little information about the linear instability, which is driving the fluctuations. A linear stability analysis of the measured potential and density profiles showed that several modes with high azimuthal mode numbers ($m \leq 15$) were unstable (Huld *et al.* 1991). This also indicates that the instability saturates mainly by nonlinear spectral cascades and not by changing (e.g. flattening) the potential and density profiles driving the fluctuations. The interpretation in terms of inverse spectral cascade is further supported by a numerical study of the nonlinear evolution of the Kelvin-Helmholtz instability on an annulus, modelling the edge region of the Q-machine plasma.

NUMERICAL RESULTS

The numerical investigations are based on the flute-mode equations for low-frequency electrostatic fluctuations (i.e. the frequencies are much smaller than the ion cyclotron frequency), which are equivalent to the Navier-Stokes equations for two-dimensional incompressible flows:

$$\partial_t \omega + (\hat{z} \times \nabla\phi) \cdot \nabla\omega = \nu \nabla^2 \omega, \tag{1}$$

where the normalized potential, ϕ, takes the role of the streamfunction with the velocity given as $\mathbf{V} = \hat{z} \times \nabla\phi$, $\hat{z}$ is the unit vector in the direction of $\mathbf{B}_0$, and the vorticity, $\omega = \nabla^2\phi$, is equivalent

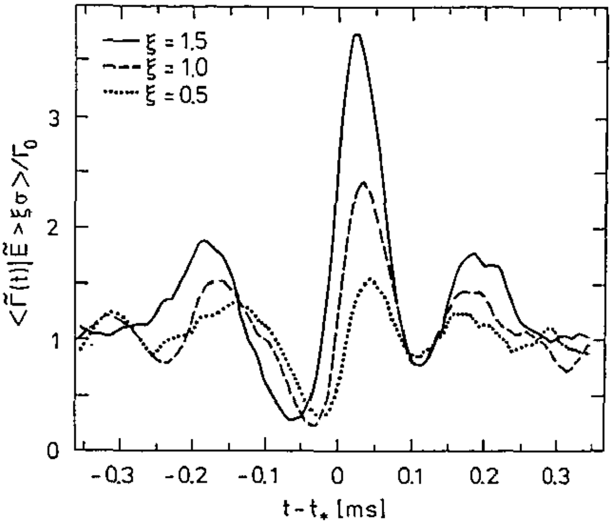

Figure 4: Conditionally averaged time varying flux $< \tilde{\Gamma}(t)|\tilde{E} \geq \xi\sigma >$ at a reference position $R = 16$ mm subject to the conditions $\tilde{E} \geq \xi\sigma$, with $\xi = 1.5$, shown with full line, $\xi = 1.0$, with dashed line, while $\xi = 0.5$ is shown with dotted line. Plasma conditions were $n_0(r = 0) = 6 \cdot 10^{14}$ m^{-3} and $B_0 = 0.275$ T.

to the charge density. Equation 1 is solved numerically in an annular geometry employing a fully de-aliased, spectral scheme, see Coutsias *et al.* (1989). The viscosity on the right hand side is introduced to suppress, in a controlled manner, the short wavelength ringing that is otherwise produced due to the finite spatial resolution. This viscosity may be thought of as representing damping due to finite Lamor radius effects. As initial condition we have set up a vorticity (charge) distribution corresponding to the averaged distribution in the experiments. This distribution corresponds to an azimuthal velocity with a jet-like radial profile, and a radial potential profile increasing from the inner boundary. The potential was kept at a constant, prescribed, value on the boundaries resulting in free slip boundary conditions, i.e. the radial velocity component vanished on the boundaries. The vorticity was set equal to zero on the boundaries. In Fig. 3 we show an example of the evolution of the potential. The flow quickly develops a growing short wave instability in correspondence with predictions from a linear stability analysis. However, this high mode-number instability evolves nonlinearly, demonstrating the inverse cascade, i.e. coalescence of like-signed vortices, and the instability saturates when the energy is condensed in large scale structures. Note that for the present profile the $m = 1$ mode is *linearly* stable, while the $m = 2$ mode is only weakly unstable. Thus, the saturated state is far from what would have been predicted by a quasi-linear analysis. This behavior agrees qualitatively with our experimental observations.

TURBULENT TRANSPORT

In order to monitor the turbulent transport we measured the density fluctuations, $\tilde{n}$, and the azimuthal electric field component, $\tilde{E}$, with closely spaced Langmuir probes. The radial plasma velocity component for both ions and electrons is well approximated by $\tilde{E}/B_0$ for these low frequency fluctuations. The local radial plasma flux is then readily obtained by multiplying the two signals electronically. Analyzing the resulting flux signal $\tilde{\Gamma}(t) = \tilde{n}(t)\tilde{E}(t)/B_0$ we can get detailed information about the nature of the plasma transport. We performed a conditional analysis of this signal, selecting time sequences which were characterized by an electric field exeeding, at a reference time, a prescribed value, E_*, which we measured in units of the rms-value, σ, of the

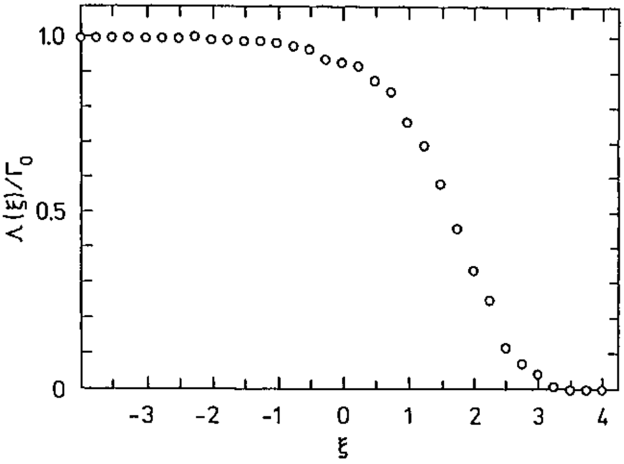

Figure 5: Normalized variation with ξ of the time-integrated conditionally averaged time varying flux $\Lambda(\xi)$. The normalizing quantity is $\Gamma_0 \approx 5 \times 10^{13}\,\mathrm{m}^{-2}\,\mathrm{sec}^{-1}$. Parameters as in Fig. 4.

record for $\tilde{E}$. Typical variations of the corresponding averaged signals are given in Fig. 4 for three values of E_*. The normalizing quantity is $\Gamma_0 =< \tilde{n}\tilde{E} > /B_0$. By time-integrating results like these for varying $E_* = \xi\sigma$, we were able to determine that fraction of the net turbulent flux, which was associated with time intervals, where coherent large scale vortices of a certain peak amplitude were present. The results of such an investigation are shown in Fig. 5. Details in the analysis and the underlying arguments were reported by Huld *et al.* (1991). We found that vortices with peak value of the associated azimuthal electric field exceeding the rms-value are responsible for approximately two-thirds of the turbulent transport. We interpret this observation as an indication of the transport in our experiment being a mixture of Brownian type diffusion and sporadic bursts. It was explicitly demonstrated that the largest amplitude eddies were actually very effective in transporting plasma out of the column, but they occured so rarely that their accumulated effect was small compared to that of their small amplitude counterparts.

Although our experiments were carried out in a specific device, it should be noted that the basic charteristics of the turbulence discussed in the present work are very similar to those of certain tokamak experiments. We therefore believe that we are dealing with universal phenomena. It is thus reasonable to expect that the basic properties of vortex-like structures generated by the Kelvin-Helmholtz instability, as described in the present paper, will be characteristic for other plasma conditions as well.

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