505 Set 6

November 27, 2012

Write a program to solve a first order system of the form $\mathbf{y}' = \mathbf{f}(t, \mathbf{y})$, $\mathbf{y}(a) = \mathbf{y}_0$ on $a \le t \le b$ using a predictor-corrector method of at least 4th order accuracy (say Adams-Basforth). Use some RK method for starting values as well as for changing stepsize. Increase or decrease stepsize based on testing $||\mathbf{u}_{j+k} - \mathbf{u}^*_{j+k}||$.

Use this code to compute some orbits of the restricted 3-body problem which in an appropriate moving dimensionless coordinate frame with fixed distance, 1, between the two massive bodies located on the x-axis with center of mass at the origin yields the equations of motion for the (small) third body at $(x(t), y(t))$ as:

$$\begin{aligned} \ddot{x} &= x + 2\dot{y} - (1-\mu)\frac{x+\mu}{[(x+\mu)^2+y^2]^{3/2}} - \mu\frac{x-1+\mu}{[(x-1+\mu)^2+y^2]^{3/2}} \quad , \\ \ddot{y} &= y - 2\dot{x} - (1-\mu)\frac{y}{[(x+\mu)^2+y^2]^{3/2}} - \mu\frac{y}{[(x-1+\mu)^2+y^2]^{3/2}} \quad . \end{aligned}$$

Use the mass ratio $\mu = 0.012277471$ and try the initial conditions

$$x_0 = 0.994 \ , \ y_0 = \dot{x}_0 = 0 \ , \ \dot{y} = -2.031732629557 \ , \ (\text{period } T = 11.124) \quad (1)$$
$$x_0 = 0.994 \ , \ y_0 = \dot{x}_0 = 0 \ , \ \dot{y} = -2.113898796645 \ , \ (\text{period } T = 5.437) \quad (2)$$

Give plots of $x(t)$ vs. $y(t)$ for $0 \le t \le 4T$ for each case. Can you find initial conditions resulting in other periodic orbits? (You must refer to literature on the restricted three body problem if you want to try this! The orbits you are asked to compute are closed orbits in the vicinity of the Lagrange points in the corotating frame. Note that the moon/earth mass ratio, 0.0123000383, is actually a little different than the value used above so the value of μ may also need to be modified if you hunt for other interesting cases in the celestial mechanics literature.)