Math. 505, Set 1.6a2

```
% script file roundoff
fprintf('difference based  vs. rationalized \n')
for n = 1:10
   m = 10^n; sum1 = 0; sum2 = 0;
   for k = 1:m
        sum1 = sum1+(1/k-1/(k+1));
   end
   ex = m/(m+1);
   fprintf('%7i %22.14f %22.14e \n',m,sum1, (sum1-ex)/ex)
end
```

```
      10      0.90909090909091 -1.22124532708767e-016
     100      0.99009900990099 -4.48530101948563e-016
    1000      0.99900099900100  5.55666623824891e-016
   10000      0.99990000999900  3.33100214078286e-016
  100000      0.99999000010001  1.26566690461516e-014
 1000000      0.99999900000105  4.69624809040781e-014
10000000      0.99999989999981 -1.95732338814647e-013
```

Math. 505, Set 1.6b

```
% script file roundoff2
fprintf('difference based  vs. rationalized \n')
for n = 1:10
   m = 10^n;
      product1 = 1;
   for k = 1:m-1
      product1 = product1*(k/(k+1));
   end
   fprintf('%9i %22.14f %22.14e \n',m,product1,m*product1-1)
end
```

```
      10      0.10000000000000  1.38777878078145e-016
     100      0.01000000000000  1.73472347597681e-016
    1000      0.00100000000000  1.08420217248550e-015
   10000      0.00010000000000 -6.77626357803440e-016
  100000      0.00001000000000 -1.23666810299128e-014
 1000000      0.00000100000000 -1.33407689192552e-014
10000000      0.00000010000000 -7.26595450066580e-014
```

Math. 505, Set 1, Problem 14 Recurrence for the exponential integral

$$Y_n = \int_0^1 t^n e^{-t} dt; n = 0,1,\cdots$$

$$Y_n = nY_{n-1} - e^{-1}; y_0 = 1 - e^{-1}$$

$$a_n = 1 - e^{-1}\sum_0^n \frac{1}{k!} = \frac{1}{e}\sum_{n+1}^{\infty}\frac{1}{k!}$$

$$Y_n = n!a_n = \frac{1}{e}\sum_{n+1}^{\infty}\frac{n!}{k!} \xrightarrow{n\to\infty} 0$$

```
% script exp_recur
n=22;
y=zeros(n,1);
c=exp(-1);
y(1)=-c+1;i=0;
fprintf('%7i %22.14f \n',i,y(1))
for i=2:n
   y(i)=(i-1)*y(i-1)-c;
   fprintf('%7i %22.14f \n',i-1,y(i))
end
```

0	0.63212055882856
1	0.26424111765712
2	0.16060279414279
3	0.11392894125692
4	0.08783632385625
5	0.07130217810980
6	0.05993362748735
7	0.05165595124002
8	0.04536816874875
9	0.04043407756730
10	0.03646133450151
11	0.03319523834515
12	0.03046341897041
13	0.02814500544389
14	0.02615063504299
15	0.02438008447347
16	0.02220191040406
17	0.00955303569759
18	-0.19592479861476
19	-4.09045061485180
20	-82.17689173820752
21	-1726.08260594352920

General solution of the recurrence involves particular solution found above plus an arbitraty multiple of a(n exploding) homogeneous solution

$$y_n = An! + Y_n$$

So, although we may start with the correct IC to pick out the bounded solution, roundoff introduces a small piece of the homo. which explodes, dominating the computation. When we compute backwards, we suppress the homogeneous part, which now is decaying (while the desired solution is growing, if only very slowly). What happens, is the solution is again composed of homogeneous and particular; however, at the starting point (n=N), where we set the solution to zero this implies that

$$y_N = AN! + Y_N = 0 \Rightarrow A = -\frac{Y_N}{N!} << 1$$

But then, the general solution is

$$y_n = Y_n - \frac{n!}{N!} Y_N$$

which gets better for smaller n:

```
% script exp_recb
  n=10,22;
  y=zeros(n,1);
  c=exp(-1);
  %y(1)=-c+1;
   for i=n-1:-1:1
       y(i)=(y(i+1)+c)/i;
fprintf('%7i %22.14f \n',i-1,y(i))
   end
```

13	0.02627710294082
12	0.03031973416248
11	0.03318326461116
10	0.03646024598024
9	0.04043396871517
8	0.04536815665407
7	0.05165594972819
6	0.05993362727138
5	0.07130217807380
4	0.08783632384905
3	0.11392894125512
2	0.16060279414219
1	0.26424111765682
0	0.63212055882826

20	0.01751806862721
19	0.01926987548993
18	0.02037627982428
17	0.02156976227754
16	0.02290877667347
15	0.02442426361531
14	0.02615358031912
13	0.02814521582075
12	0.03046343515325
11	0.03319523969372
10	0.03646133462411
9	0.04043407757955
8	0.04536816875011
7	0.05165595124019
6	0.05993362748738
5	0.07130217810980
4	0.08783632385625
3	0.11392894125692
2	0.16060279414279
1	0.26424111765712
0	0.63212055882856