The University of New Mexico

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504 '09-Project

"To build an efficient symmetric eigensolver"

We want to write a fast eigensolver for moderate-sized symmetric matrices. It should be able to give efficiently either a selected few or all eigenvalues and, optionally, eigenvectors. All tricks you know must be incorporated, with key objective to perform the calculation in the most efficient way possible:

- Conversion to (symmetric = tridiagonal) Hessenberg form.
- Efficient/stable solution of triangular system (Rayleigh quotient iteration step) using either sparse LU with pivoting or Givens QR.
- for the case when all eigenvalues are required, follow the steps given in problem 29.1 of Trefethen and Bau. For the case where only one eigenpair is sought, use Rayleigh quotient iteration.
- if the largest eigenvalue is sought, use Gershgorin estimates to get a good upper bound for its value before starting the iteration. You must ensure that you did not converge to a lesser eigenvalue.
- Document the flops as well as the convergence rate of your algorithms. Employ random symmetric matrices of size $2^k, k = 4, \ldots, 12$ and give the number of iterations required by RQI method to find the largest eigenvalue. Also give the number of iterations required by QR to converge. Verify that the eigenvalue found by RQI method is indeed the same as the largest eigenvalue found by QR method.
- Do you encounter any problems with the convergence of RQI? Discuss

- If you can use f95 or C you may do so, however if you can only use Matlab, then you may have to restrict the size of the problem to get convergence in finite time.