

Math. 466  
Spring '97

Problem Set V

February 17

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Due: Thu, Feb. 27, 1997

Evaluate the following integrals using contour integration.

(1.)  $\int_0^\pi \frac{d\theta}{(a+\cos\theta)^2}$ ,  $a > 1$   $\left( = \frac{\pi a}{(a^2-1)^{3/2}} \right)$

(2.)  $\int_1^\infty \frac{x dx}{(x^2+4)\sqrt{x^2-1}}$  (positive square root is indicated)

Hint: let  $x = 1/t$  and convert to an integral over  $[0, 1]$ .

(3.) (a)  $\int_0^\infty \frac{dx}{x^3+x^2+x+1}$  \* (b)  $\int_0^\infty \frac{dx}{x^3-1}$  (a principal value integral)

(4.)  $\int_0^\infty \frac{\sqrt{x} dx}{x^2+1}$

(5.)  $\int_{-\infty}^\infty \frac{x e^{ikx}}{x^2+1} dx$  ( $k > 0$ )

\* (6.)  $\int_{-\infty}^\infty \frac{e^{-i\omega x}}{\cosh x} dx$  ( $\omega$  is a real number)

(Hint: Integrate  $\frac{e^{-i\omega z}}{\cosh z}$  around the rectangle with corners at  $R$ ,  $R+i\pi$ ,  $-R+i\pi$ ,  $-R$  and let  $R \rightarrow \infty$ ).