466 '07 (E.A. Coutsias)-HOMEWORK 9

due: Tuesday, November 20, 2007

November 25, 2007

1. Find (a) Fourier (b) Fourier cosine (c) Fourier sine series of period 4 (period 2 for case a) for $f(x)$ where it is given that
$$f(x) = \begin{cases} 5 & \text{for } 0 < x < 1 \\ 0 & \text{for } 1 < x < 2 \end{cases} .$$
Sketch the function to which each series converges in $-4 \le x \le 4$. Account for every point in $-4 \le x \le 4$.

2. Solve
$$\begin{aligned} u_{xx} + u_{yy} = 0 \quad &, \quad 0 < x < b \ , \ y > 0 \\ u(0, y) = u(b, y) = 0 \quad &, \quad u(x, 0) = f(x) \ , \ u(x, y) \text{ bounded as } y \to \infty \ . \end{aligned}$$

3. Tidal waves approaching a long, straight beach are described by
$$\frac{\partial}{\partial x}\left(h(x)\frac{\partial z}{\partial x}\right) = \frac{1}{g}\frac{\partial^2 z}{\partial t^2} \ ,$$
where $z(x,t)$ is the elevation measured from the undisturbed free surface, $h(x)$ is the depth of the water at distance x from the beach, and g is the acceleration of gravity. Show that if the bottom depth increases linearly with distance from the beach (i.e. take $h(x) = kx$ where $k =$ constant) then tidal waves of frequency $\frac{\omega}{2\pi}$ exist for which
$$z(x,t) = AJ_0\left(\frac{2\omega}{\sqrt{kg}}x^{1/2}\right)\cos\omega t \ .$$
Show that the wavelength of these waves increases with increasing distance from the beach.
(*Hint:* use the asymptotic form of $J_0(x)$ for large x.)