# 466 '07 (E.A. Coutsias)-HOMEWORK 7

due: Friday, November 2, 2007

October 24, 2007

1. Consider the Legendre equation
$$\left(1-x^2\right)y''-2xy'+\alpha(\alpha+1)y=0\ .$$
Find the power series about the origin for each of the two linearly independent solutions of the equation. Show that if $\alpha$ is zero or a positive ieven integer, $\alpha=2n$, the series expansion for one of these solutions reduces to a polynomial of degree $2n$ containing only even powers of $x$ and show that corresponding to $a=0,\ 2,\ 4$ these polynomials are $1,\ 1-3x^2,\ 1-10x^2+(35/3)x^4$. Show that if $\alpha$ is a positive, odd integer, $\alpha=2n+1$, the series for the other solution reduces to a polynomial of degree $2n+1$, containing only odd powers of $x$ and that corresponding to $a=1,\ 3,\ 5$ these polynomials are $x,\ x-(5/3)x^3$, $x-(14/3)x^3+(21/5)x^5$. The Legendre polynomial $P_n$ is defined as the polynomial solution of the Legendre equation with $\alpha=n$ which satisfies $P_n(1)=1$ Find $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$.

2. Find the first three terms in linearly independent solutions of
$$y''-xy=0\ ,$$
in powers of $(x-1)$.

3. Determine the singular points of the equations
$$\begin{array}{ll}(a) & zu''+u'-u=0\\ (b) & zu''-(1+z)u'+2(1-z)u=0\\ (c) & (2z+z^3)u''-u'-6zu=0\end{array}$$
In each case determine the exponents of the regular singular points (the point $z=\infty$ should also be considered).

4. (*Optional*) For equations $3a$, $3b$, find two independent series solutions valid near $z = 0$.

5. Determine a particular solution of the following equations in the form of a series valid near $x = 0$. In each case obtain the first three nonvanishing terms

$$(a) \qquad y'' + y = e^x$$
$$(b) \qquad y'' + xy = 1$$
$$(c) \qquad xy'' - y = x$$
$$(d) \qquad x^2y'' + y = \frac{e^x}{\sqrt{x}}$$