466 '07-HOMEWORK 1

due: Thursday, August 30, 2007

August 21, 2007

1. Is $u(x,y) = x\cos y\cosh x + y\sin y\sinh x$ the real part of an analytic function? Answer the same question for $u(x,y) = x\cos y\cosh x - y\sin y\sinh x$.

2. Derive the Cauchy-Riemann equations in polar coordinates and determine if $re^{\cos\theta}$ is a harmonic function.

3. If $f(z) = u(x,y) + iv(x,y)$, show that the curves $u(x,y) = \text{const}$ and $v(x,y) = \text{const}$ are orthogonal at every point where $f'(z)$ exists and is not zero. Sketch the families of curves for the case where $f(z) = u + iv = z^2$ and comment on the structure near the origin of the z-plane.

4. Establish the formula

$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z} \ ,$$

for the sum of a finite geometric series. Then, derive the formulas

$$1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin\left[\left(n + \frac{1}{2}\right)\theta\right]}{2\sin\frac{\theta}{2}}$$

$$\sin\theta + \sin 2\theta + \cdots + \sin n\theta = \frac{1}{2}\cot\frac{\theta}{2} - \frac{\cos\left[\left(n + \frac{1}{2}\right)\theta\right]}{2\sin\frac{\theta}{2}}$$

5. Write the following complex numbers in the form $a + ib$:

   (a) $(1 + 2i)^2$

   (b) $(1 + 2i)^9$

6. Find each of the roots and locate them geometrically

   (a) $\left(-2\sqrt{3} - 2i\right)^{1/4}$

   (b) $(-1 + i)^{1/3}$

7. Find all the values of z such that $z^5 = 32$.

8. Find all the values of i^i.

9. ($\tan^{-1} :=$ arctan)

   (a) Show that

   $$\tan^{-1} z = k\pi + \frac{1}{2i}\mathrm{Log}\left(\frac{1 + iz}{1 - iz}\right)$$

   where $k = 0, \pm 1, \pm 2, \ldots$

   (b) Find the values of $\tan^{-1}(1 - 2i)$.