Homework 3
MA/CS 375, Fall 2005
Due October 21

This homework will count as part of your grade so you must work independently. It is permissible to discuss it with your instructor, the TA, fellow students, and friends. However, the programs/scripts and report must be done only by the student doing the project. Please follow the guidelines in the syllabus when preparing your solutions.

1. Provide an upper bound of the Lagrange interpolation error for
$$f(x) = \sin(2x), \quad \frac{\pi}{4} \le x \le \frac{3\pi}{4},$$
with 7 equally spaced nodes:
$$x_k = \frac{\pi}{4} + \frac{k\pi}{12}, \quad k = 0, \ldots, 6.$$
Plot the interpolant (use polyfit) and compute the maximum error by evaluating f and the interpolant on 100 equally spaced points. How do the results compare with your upper bound?

2. The following is a table of the measured flame speeds at standard conditions for ethylene-air mixtures as a function of the volume percentage of the ethylene (from Linnett and Hoare, *Third Symposium on Combustion, Flame and Explosion Phenomena*):

Volume % ethylene	4.60	5.19	5.97	6.80	7.52	8.50	9.32	10.12
V in cm/s	35.1	42.5	53.7	67.5	71.7	68.0	53.3	28.6

Use both piecewise linear interpolation and spline interpolation to predict the flame speed for two ethylene-air mixtures which are 7.91% and 9.0% ethylene by volume, respectively . The measured speeds at these volumes are 70.9 cm/s and 61.1 cm/s respectively. Which predictions better matched the measurements?

3. Use **fft** to decode a time series of 7 pulses (the signal of a phone number dialing) Each person will be assigned a different number, but they can all be downloaded from the web page. In particular, use the signal in the file chosen from tele_1.wav ... tele_9.wav where the number corresponds to the last digit of your ID number. (If it is zero use the last nonzero digit.) You will need this information which lists the two dominant frequencies in Hz corresponding to each button on the touch-tone keyboard. Additional information (which you may find useful in decoding some of these signals), can be found in the file "numbers", available on the web page.

(in Hz)	1209	1336	1477
697	1	2	3
770	4	5	6
852	7	8	9
941	*	0	#

4. Evaluate the function $f(x) = \cos(2\pi x)$ at 17 equispaced nodes in $[-1, 1]$. To the data at each node add random perturbations uniformly distributed in $[-10^{-3}, 10^{-3}]$. Use **polyfit** with the perturbed data to compute least squares polynomial approximations of degrees 1-16. For each of the 16 approximations compute the normalized RMS difference between the approximating polynomial and f at 100 equally spaced points in $[-1, 1]$. (The normalized RMS difference between the length 100 vectors $y1$ and $y2$ is computed by $\text{norm}(y1 - y2)/100$.) Plot the differences versus the polynomial degree. Which degree yields the best results? Suggest an explanation.

5. Show that the regression line passes through the point whose abcissa is the average of $\{x_i\}$ and whose ordinate is the average of $\{f(x_i)\}$. (Give a more detailed answer than in the text's solutions.)