Homework 2
MA/CS 375, Fall 2005
Due September 30

This homework will count as part of your grade so you must work independently. It is permissible to discuss it with your instructor, the TA, fellow students, and friends. However, the programs/scripts and report must be done only by the student doing the project. Please follow the guidelines in the syllabus when preparing your solutions.

1. Use the bisection method, Newton's method, and the MATLAB function fzero to compute a positive real number x satisfying:
$$\sinh x = \cos x.$$
For each of the three methods use a tolerance of 10^{-8}. List your initial approximation (or interval in the case of bisection) and the number of iterations needed. Also print at least nine digits of the approximate roots.

2. Use the bisection method, Newton's method, and the MATLAB function fzero to compute all three real numbers x satisfying:
$$5x^2 - e^x = 0.$$
For each of the three roots and each of the three methods use a tolerance of 10^{-8}. List your initial approximation (or interval in the case of bisection) and the number of iterations needed. Also print at least nine digits of the approximate roots.

3. Consider the use of Newton's method to solve:
$$e^{-x^2} - 1 = 0,$$
for the root $x = 0$. Reformulate the method as a fixed point iteration and find the rate at which it will converge. (Hint: use l'Hopital's rule to evaluate the derivative of the iteration function as $x \to 0$.)

4. Consider a 4-bar planar linkage ABCD where the four rods have lengths $AB = a_1$, $BC = a_2$, $CD = a_3$ and $DA = a_4$. If we introduce the angles $\alpha = \angle ABC$ and $\beta = \pi - \angle DAB$, we have the system described in the text, **problem 2.3**, p. 38 (see also Fig. 2.1 in the text). This is called a planar linkage system, and it can exist in various shapes. As the angle α is varied that will result in turn in changes in the angle β; the two angles are related by equation (text, eq. 2.2):
$$\frac{a_1}{a_2}\cos(\beta) - \frac{a_1}{a_4}\cos(\alpha) - \cos(\beta - \alpha) = -\frac{a_1^2 + a_2^2 - a_3^2 + a_4^2}{2a_2a_4}$$
Apply Newton's method to solve this problem for $\alpha \in [0, 2\pi]$ with a tolerance of 10^{-6}. Assume that the lengths of the rods are $a_1 = 10$, $a_2 = 2$, $a_3 = 7$, $a_4 = 6$. For this arrangement, and to each value of α in the given range there correspond two possible values of β. Use an initial value of $\beta = -\pi/2$, which ensures that you will get a solution for β in the same contiguous range. Plot the position of the mid-point of the rod CD as α takes values in $[0, 2\pi]$.