Math. 375, Fall 2005
2. Matrices and Vectors

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$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

Square matrix: size nXn

A = [1, 2, 3; 4, 5, 6];

Rectangular matrix, size 2X3

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

Concepts from Linear Algebra

- Eigenvalues and Eigenvectors

  eig(A)
- Solution of Linear Systems

  x = A/b
- Linear Independence
- Determinants

  d = det(A)
- Matrix Inverse

  B = inv(A)

Special Matrices and Commands

- Identity: E = eye(n)
- Zero matrix: zeros(m,n)
- Inverse: inv(A)
- Matrix sum: A+B (both must be mXn)
- Matrix product: C = A*B

  (A is mXk, B is kXn and C is mXn)
- Dimensions: size(A) = (m,n)
- Determinant: det(A)

```
% Banded Matrices
% script tridiag
A=[ 0  1  2  3;
   -1  0  1  2;
   -2 -1  0  1;..
   -3 -2 -1  0;
   -4 -3 -2 -1];
v=diag(A,-1)
C=diag(v,3)
B1 = tril(A,1)
B2 = triu(A,-1)
T1 =-triu(tril(ones(6,6),1),-1)+3*eye(6,6)
T2 =-diag(ones(5,1),-1)+diag(ones(6,1),0)…
    -diag(ones(5,1),1)
T3 = toeplitz([2;-1;zeros(4,1)])
```

Matrix displays

- spy(A)
- image(A)
- mesh(A)
- pltmat(A,'name',colormap,font)
- load gatlin, image(X);colormap(map),…
- bar3(A)
- >>demo: graphs & matrices, intro
- whos shows workspace

Arrays: vectors

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

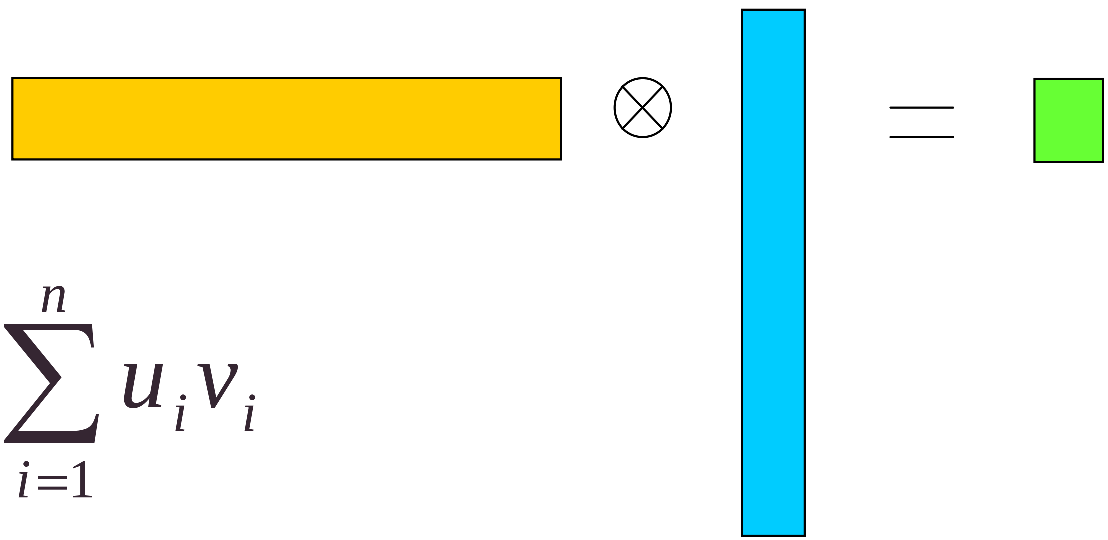
$$\vec{x}' = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}$$

x is a **column** vector while its transpose, **x**', is a **row** vector

Basic vector/matrix operations

Inner product: **u’*v = dot(u,v) ;**

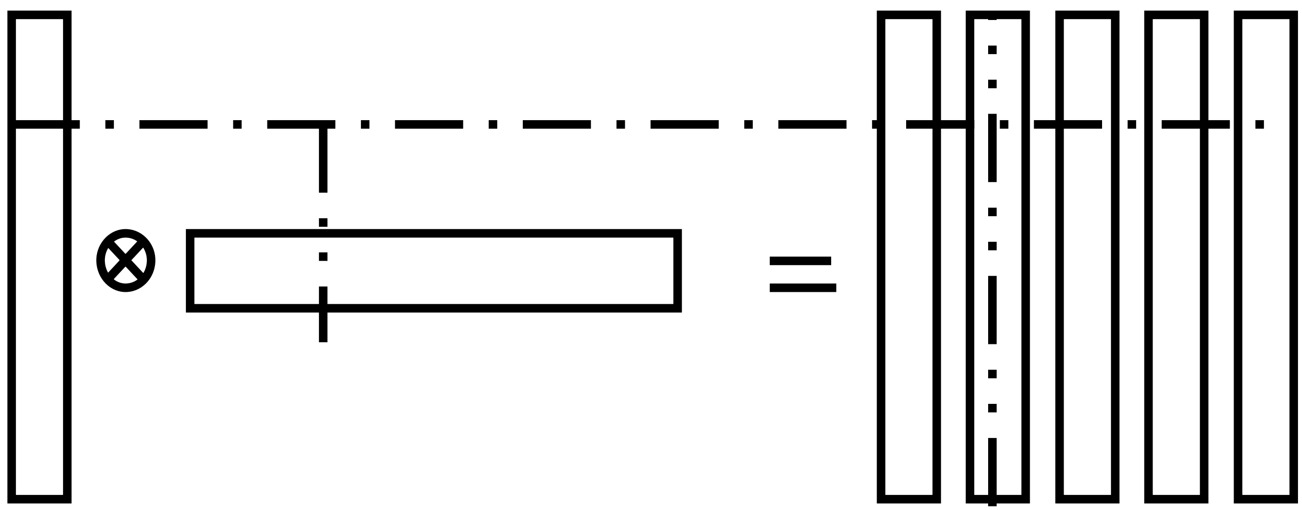
```
sum=0;
for i=1:length(u)
    sum = sum+u(i)*v(i);
end
```



$$< u, v >= u'v = \sum_{i=1}^{n} u_i v_i$$

Outer Product: **u*v';**

```
for i=1:length(u)
    for j=1:length(v)
        uv(i,j) = u(i)*v(j);
    end
end
```



$$u \otimes v = uv' = (u_i v_j)$$

Pointwise operations

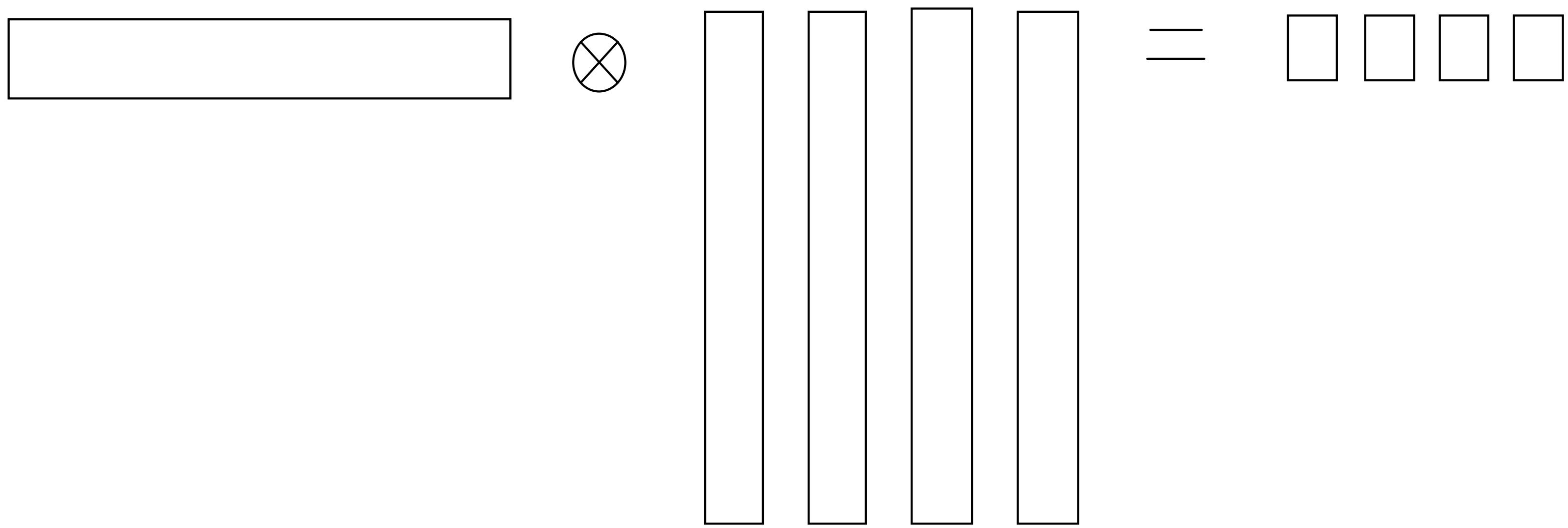
$$x = [x_1 \quad x_2 \quad x_3]$$

$$y = [y_1 \quad y_2 \quad y_3]$$

$$x.*y = [x_1 y_1 \quad x_2 y_2 \quad x_3 y_3]$$

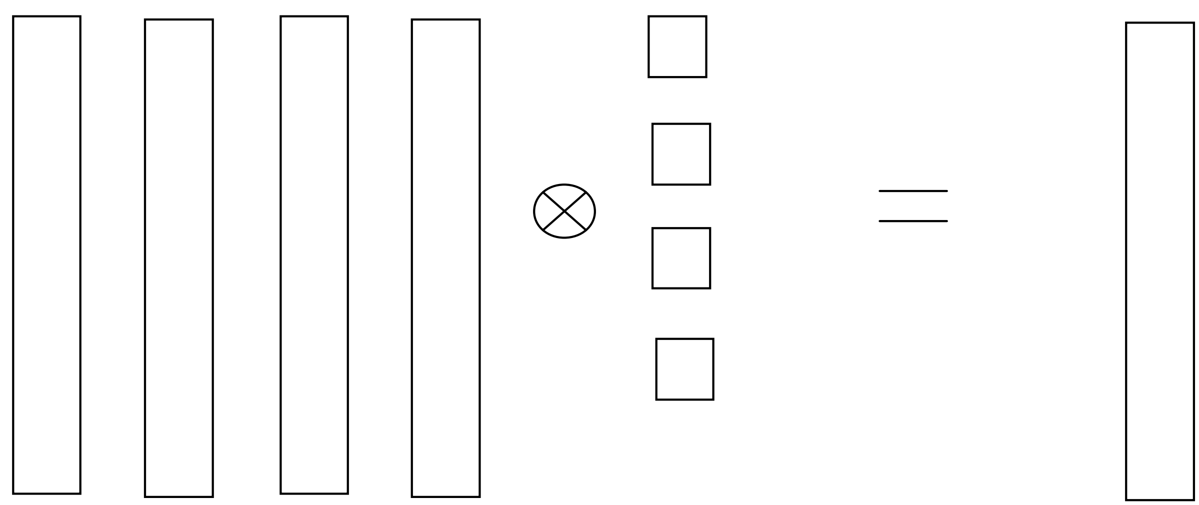
$$x.\wedge 2 = [x_1^2 \quad x_2^2 \quad x_3^2]$$

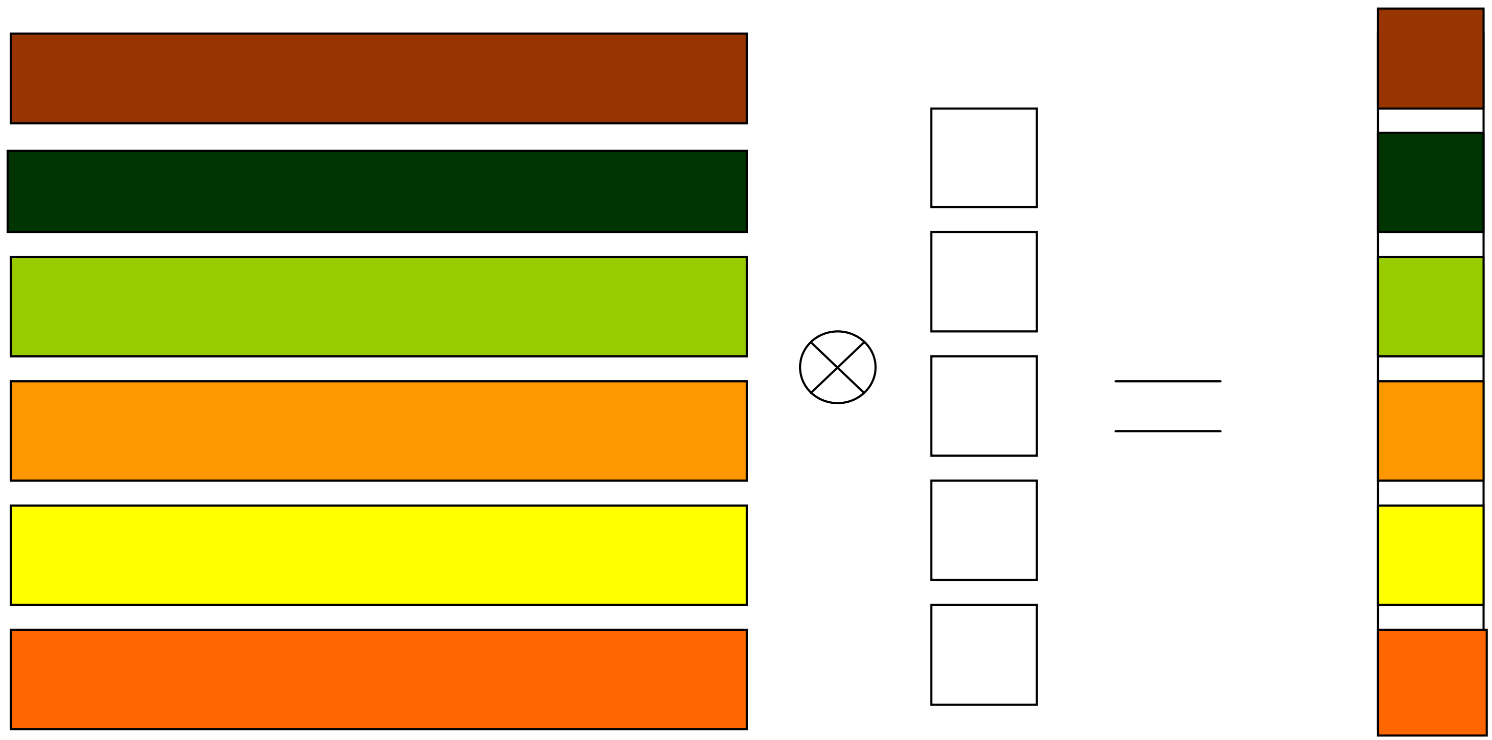
1xN row X Nx4 matrix = 1x4 row



1

Nx4 matrix X 4x1 column = Nx1 column

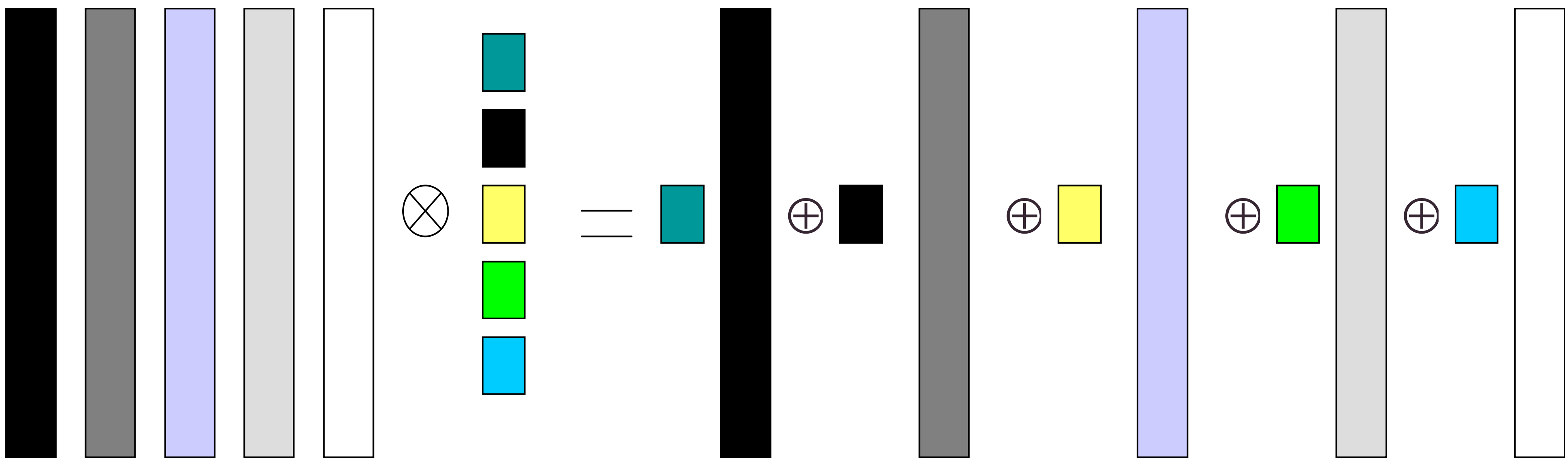




Matrix-Vector multiplication:

The ROW picture

Matrix-Vector Multiplication:
The COLUMN picture



II.VECTOR OPERATIONS:

(1) vector: scale, add, subtract

 (scalar*vector):

  2*[10 20 30] = [20 40 60]

(2) POINTWISE vector:

  multiply, divide, exponentiate

 (pointwise vector * vector):

  [2 3 4].*[10 20 30] = [20 60 120]

  (for pointwise operations,
   dimensions must be identical!)

(3) Vector linear combos: matrix*vector, A*x

(4) size(A) gives array dimensions

  (array, can be used to index other arrays)

  length(x) gives vector dimension

(5) for k = 1:n

```
for k = 1:n
        (operations)
    end
```

  (loop around n times)