# Numerical Methods for ODEs

Euler

Augmented Euler

# euler

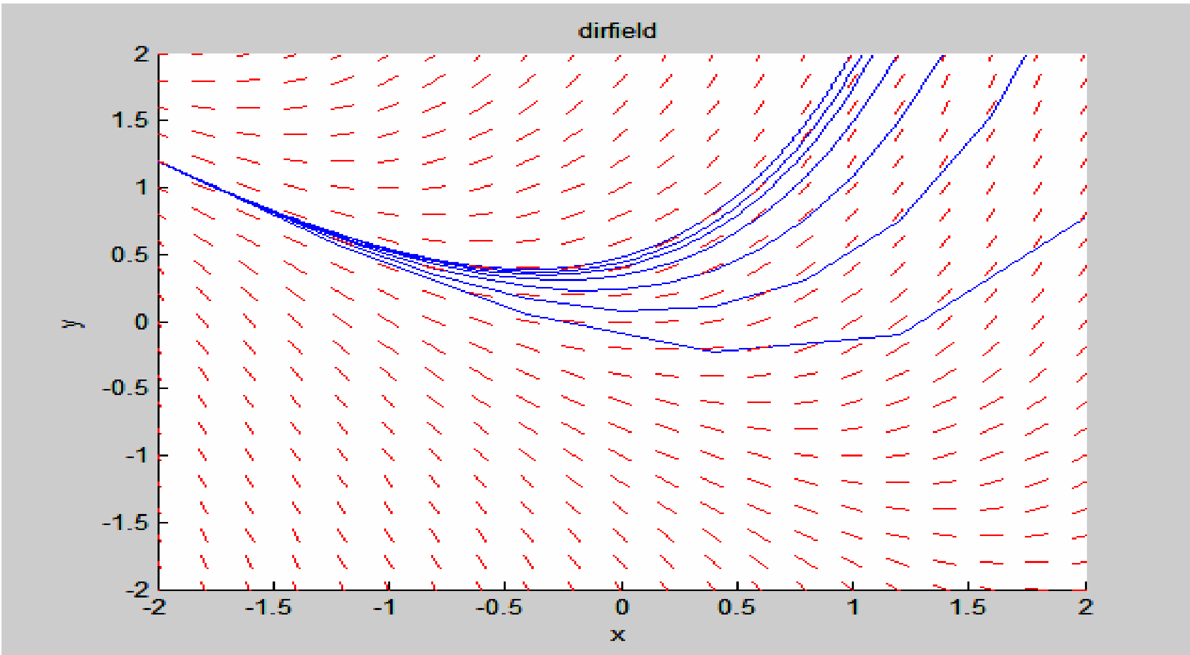
```
function [X, Y] = euler(x,y,x1,n)
  h = (x1-x)/n;
  X = x;
  Y = y;
  for i = 1:n
      k = f(x,y);
      x = x + h;
      y = y + h*k;
      X = [X; x];
      Y = [Y; y];
  end
  hold on
  plot(X,Y)
  axis([-2 2 -2 2])
  X1 = linspace(-2,2,21); Y1 = linspace(-2,2,21);
  hold on
  df(@f,X1,Y1,'x','y','dirfield','r',0);

  function yp = f(x,y)
    yp = x+y;
```

# An example of euler's method

$$y' = x + y,$$

$$y(-2) = 1.2$$

$$n = 5,10,20,40,80,160,320$$



## Different step size: varying answers

- Notice how decreasing the stepsize in the previous example produces smoother solution curves. These keep changing as the stepsize decreases, but do approach a limit for very small (approx 4x10^-4) stepsize h. The rate of convergence of Euler’s method is linear (i.e. the error decreases proportionally to h).

# Improved Euler

- function [X, Y] = impeuler(x,y,x1,n)
- h = (x1-x)/n;
- X = x;
- Y = y;
- for i = 1:n
- k1 = f(x,y);
- k2 = f(x+h, y+h*k1);
- k = (k1 + k2)/2;
- x = x + h;
- y = y + h*k;
- X = [X; x];
- Y = [Y; y];
- end

```
hold on
  plot(X,Y)
  axis([-2 2 -2 2])
  X1 = linspace(-2,2,21); Y1 = linspace(-2,2,21);
  hold on
  df(@f,X1,Y1,'x','y','dirfield','r',0);

  function yp = f(x,y)
    yp = x+y;
```

# Same calculation using Euler's improved method

Notice how the curves are much vloser together, even for 10 point resolution