Solutions, 316-IX

February 16, 2003

1 Problem 4.8.4

Find a particular solution to the DE

$$x''(t) - x(t) = 3e^{-2t} \ .$$

Solution:
The characteristic equation is $r^2 - 1 = 0$ with roots $r = \pm 1$. Since -2 is not a root, try

$$\begin{aligned} x_p(t) &= Ae^{-2t} \\ x_p'(t) &= -2Ae^{-2t} \\ x_p''(t) &= 4Ae^{-2t} \end{aligned}$$

so that

$$x'' - x = (4A - A)e^{-2t} = 3e^{-2t}$$

i.e.

$$A = 1 \to x_p(t) = e^{-2t} \ .$$

2 Problem 4.8.35

Determine the form of a particular solution to the DE

$$x'' - x' - 2x = e^t \cos t - t^2 + t + 1 \ .$$

Solution:
The characteristic equation is $r^2 - r - 2 = 0$ with roots $r = 1, -2$. Since 0 and $1 + i$ are not roots, try

$$\begin{aligned} x_p(t) &= e^t (A \cos t + B \sin t) + Ct^2 + Dt + E \\ x_p'(t) &= e^t \left[(A + B) \cos t + (-A + B) \sin t \right] + 2Ct + D \\ x_p''(t) &= e^t \left[(2B) \cos t + (-2A) \sin t \right] + 2C \end{aligned}$$

so that

$$x'' - x' - 2x = e^t \cos t - t^2 + t + 1$$

i.e.

$$e^t \left[(B - 3A) \cos t + (-A - 3B) \sin t \right] - 2Ct^2 - 2(C + D)t + (2C - D - 2E) = e^t \cos t - t^2 + t + 1$$

i.e.

$$\begin{aligned} B - 3A &= 1 \\ -A - 3B &= 0 \\ -2C &= -1 \\ -2(C + D) &= 1 \\ 2C - D - 2E &= 1 \end{aligned}$$

and solving:

$$A = -3/10 \ , \ B = 1/10 \ , \ C = 1/2 \ , \ D = -1 \ , \ E = 1/2$$

and

$$y_p(t) = -\frac{3}{10} e^t \cos t + \frac{1}{10} e^t \sin t + \frac{1}{2} t^2 - t + \frac{1}{2} \ .$$

3 Problem 4.8.36

Determine the form of a particular solution to the DE

$$y'' + 5y' + 6y = \sin x - \cos 2x \ .$$

Solution:
The characteristic equation is $r^2 + 5r + 6 = 0$ with roots $r = -2, -3$. Since i and $2i$ are not roots, try

$$y_p(t) = A\cos x + B\sin x + C\cos 2x + D\sin 2x \ .$$

4 Problem 4.8.45

Find a particular solution to the higher order DE

$$y''' - y'' + y = sinx \ .$$

Solution:
The characteristic equation is $r^3 - r^2 + 1 = 0$. Since i is not a root, the solution has the form

$$y_p(t) = A\cos x + B\sin x \ .$$

5 Problem 4.8.48

Find a particular solution to the higher order DE

$$y^{iv} - y'' - 8y = sinx \ .$$

Solution:
The characteristic equation is $r^4 - r^2 - 8 = 0$. Since i is not a root, the solution has the form

$$y_p(t) = A\cos x + B\sin x \ .$$