Decay, 316-XXVII

May 4, 2003

1 Problem 12.4.13

Use the energy function to assist in drawing the phase plane diagram for the given non-conservative system

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x - x^3 = 0 \; . \tag{1}$$

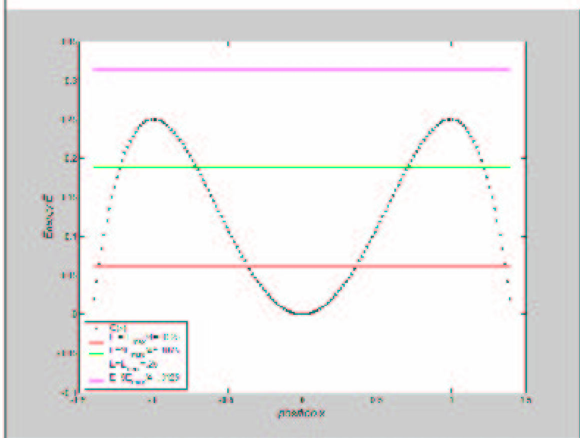
Solution:

The basic derivation in the problem: move the dissipation term to the rhs. and multiply both sides by dx/dt to show (for appropriate $G(x)$):

$$\frac{dE}{dt} := \frac{d}{dt}\left[\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + G(x)\right] = -\left(\frac{dx}{dt}\right)^2 \; .$$

That means that the energy can only decay.

We first ignore the dissipation term and draw the phase portrait of the corresponding conservative system. Then the actual orbits will follow paths in the direction of decreasing energy.

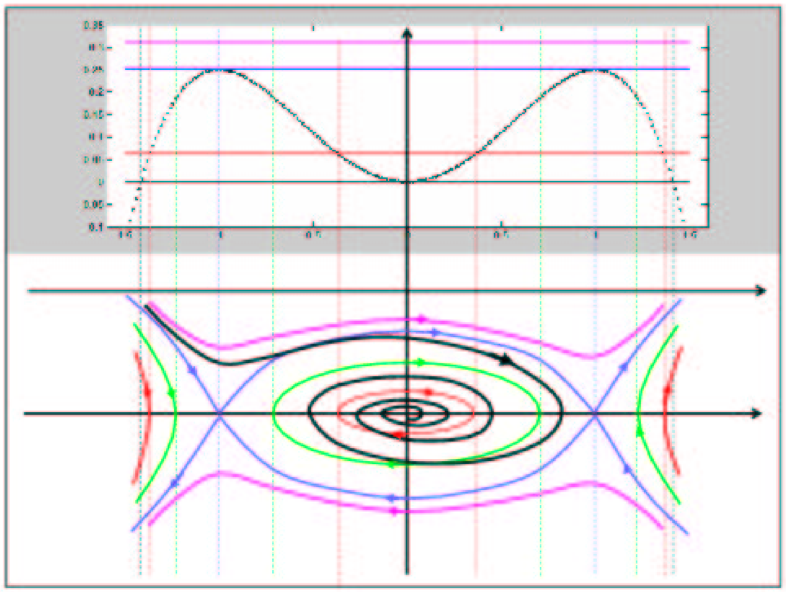

Energy levels used in graph

1

Plot of $G(x)$ together with several energy "levels". Given a value of E, a phase orbit with that energy can only exist for positions where $G(x) \leq E$. Since the difference

$$E - G(x) = \frac{1}{2}\left(\frac{dx}{dt}\right)^2$$

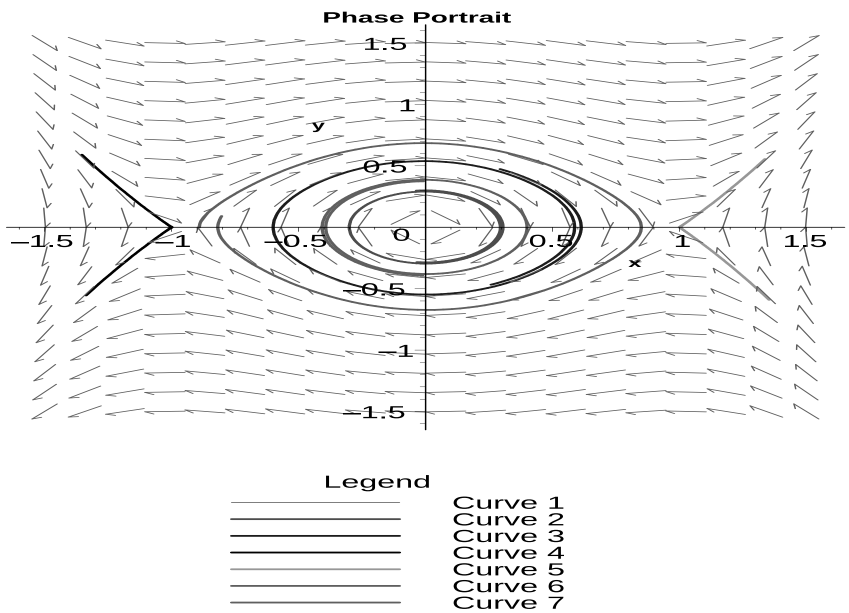
equals the kinetic energy, it is never negative (or the velocity would become imaginary!).



3

Using the potential energy diagram to draw the phase plot. The vertical axis on the bottom graph is velocity, while at the top total energy. The horizontal axis is displacement.

```
> plot1 :=
> DEplot([diff(x(t),t)=y(t),diff(y(t),t)=-x(t)*(1-x(t)^2)-.01*y(t)],
> [x(t),y(t)], t=-5..5, scene=[x(t),y(t)],
> [[x(0)=-.6,y(0)=0],[x(0)=-.3,y(0)=0],[x(0)=.4,y(0)=0],[x(0)=.85,y(0)=0
> ],[x(0)=1.0005,y(0)=0],[x(0)=-1.0005,y(0)=0]], x=-1.5..1.5,
> y=-1.5..1.5, linecolor=[red,blue,black,green,orange,orange],
> arrows=SMALL, method=rkf45, stepsize=0.05):
> display({plot1}, title="Phase Portrait", titlefont = [HELVETICA,
> BOLD, 12], labels = ["x","y"], labelfont = [HELVETICA, BOLD,
10]);
```



What the phase plot would look like if the dissipation term had a coefficient equal to .01 (instead of 1).