Solutions, 316-XIX

April 10, 2003

7.8¡13,15*,21,23*,24,29*¿ CAUTION: there may be errors!!!

1 Problem 7.8.15

Solve the given IVP:

$$y'' + 2y' - 3y = \delta(t-1) - \delta(t-2) \ ; \ y(0) = 2 \ , \ y'(0) = -2 \ .$$

Solution:

$$\begin{aligned}
\mathcal{L}\{y'' + 2y' - 3y\} &= \mathcal{L}\{\delta(t-1) - \delta(t-2)\} \Rightarrow \\
(s^2 + 2s - 3)Y(s) - 2s + 2 - 2 &= e^{s-1} - e^{s-2} \ .
\end{aligned}$$

Solving for $Y(s)$ (and using the partial fractions expansion $1/(s^2 + 2s - 3) = 1/4(1/(s-1) - 1/(s+3))$ we have

$$\begin{aligned}
Y(s) &= \frac{2s + e^{s-1} - e^{s-2}}{s^2 + 2s - 3} \\
&= \frac{1}{4}\left(2s + e^{s-1} - e^{s-2}\right)\left(\frac{1}{s-1} - \frac{1}{s+3}\right) \\
&= \frac{1}{4}\frac{2(s-1) + 2 + e^{s-1} - e^{s-2}}{s-1} - \frac{1}{4}\frac{2(s+3) - 6 + e^{s-1} - e^{s-2}}{s+3} \\
&= \frac{1}{2} + \frac{1}{4}\frac{2 + e^{s-1} - e^{s-2}}{s-1} - \frac{1}{2} + \frac{1}{4}\frac{6 - e^{s-1} + e^{s-2}}{s+3} \\
&= \frac{1}{2}\frac{1}{s-1} + \frac{3}{2}\frac{1}{s+3} + \frac{1}{4}\left(e^{s-1} - e^{s-2}\right)\left(\frac{1}{s-1} - \frac{1}{s+3}\right)
\end{aligned}$$

and inverting the transforms we find:

$$y(t) \quad = \quad \frac{1}{2}e^t + \frac{3}{2}e^{-3t} + \frac{1}{4}\left(f(t-1)u(t-1) - f(t-2)u(t-2)\right)$$

where

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s-1} - \frac{1}{s+3}\right\} = e^t - e^{-3t} \ .$$

Then

$$f(t-1) = e^{t-1} - e^{-3(t-1)} \ ,$$

and

$$f(t-2) = e^{t-2} - e^{-3(t-2)} \ .$$

Putting it all together:

$$y(t) = \begin{cases} \frac{1}{2}e^t + \frac{3}{2}e^{-3t} & , \quad 0 \le t < 1 \\ \frac{1}{2}e^t + \frac{3}{2}e^{-3t} + \frac{1}{4}e^{t-1} - \frac{1}{4}e^{-3(t-1)} & , \quad 1 \le< t < 2 \\ \frac{1}{2}e^t + \frac{3}{2}e^{-3t} + \frac{1}{4}e^{t-1} - \frac{1}{4}e^{-3(t-1)} - \frac{1}{4}e^{t-2} + \frac{1}{4}e^{-3(t-2)} & , \quad 2 \le t < \infty \ . \end{cases}$$

2 Problem 7.8.23

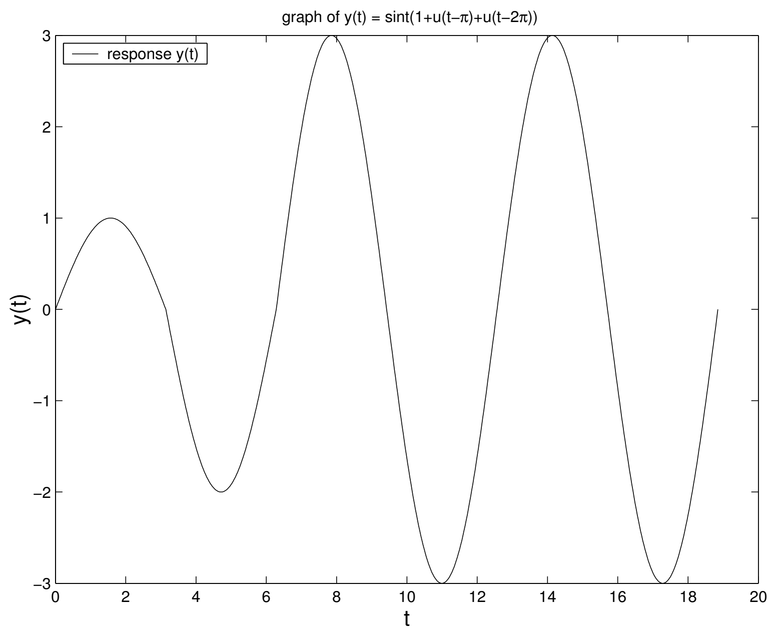
Solve the IVP and sketch the solution

$$y'' + y = -\delta(t-\pi) + \delta(t-2\pi) \; ; \; y(0) = 0 \; , \; y'(0) = 1 \; .$$

Solution:

$$\begin{aligned} s^2Y - 1 + Y &= -e^{-\pi s} + e^{-2\pi s} \Rightarrow \\ (s^2+1)Y &= 1 - e^{-\pi s} + e^{-2\pi s} \Rightarrow \\ Y &= \frac{1}{s^2+1} - \frac{e^{-\pi s}}{s^2+1} + \frac{e^{-2\pi s}}{s^2+1} \Rightarrow \\ y(t) &= \sin t - u(t-\pi)\sin(t-\pi) + u(t-2\pi)\sin(t-2\pi) \end{aligned}$$

$$y(t) = \begin{cases} \sin t & , \; 0 \le t < \pi \; , \\ 2\sin t & , \; \pi \le t < 2\pi \; , \\ 3\sin t & , \; 2\pi \le t < \infty \end{cases} \; .$$



for i = 1:201
x23(i) = sin((i-1)*pi/100);

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end
for i = 102:201
x23(i) = 2*x23(i);
end
for i = 202:601
x23(i) = 3*x23(i);
end
t = linspace(0,1,31);
z = linspace(0,pi,201);
plot(z,x24,'-')
xlabel('t','Fontsize',14)
ylabel('x(t)','Fontsize',14)
title(' graph of x(t) = cos3t(1-u(t-π/2))')
legend('response x(t)',1)
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3 Problem 7.8.29

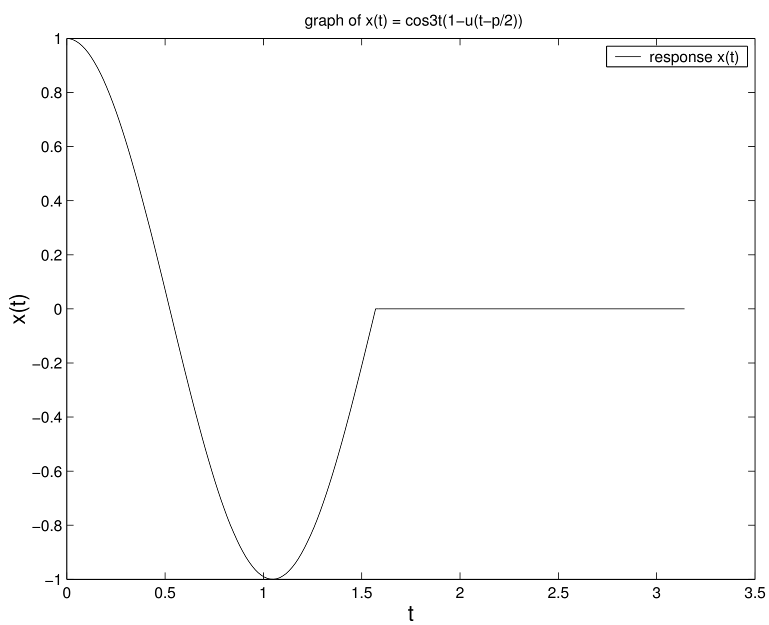
A mass attached to a spring is released from rest 1 m below the equilibrium position for the spring-mass system and it begins to vibrate. After $\pi/2$ sec, the mass is struck by a hammer exerting an impulse on the mass. The system is governed by the symbolic IVP:

$$\frac{d^2x}{dt^2} + 9x = -3\delta\left(t - \frac{\pi}{2}\right) \; ; \; x(0) = 1 \; , \; \frac{dx}{dt}(0) = 0 \; ,$$

where $x(t)$ denotes the displacement from equilibrium at time t. What happens to the mass after it is struck?

Solution:

$$\begin{aligned} s^2X - s + 9X &= -3e^{-s\pi/2} \\ X &= \frac{s}{s^2+9} - \frac{3}{s^2+9}e^{-s\pi/2} \Rightarrow \\ x = \cos 3t - \sin 3(t - \frac{\pi}{2})u(t - \frac{\pi}{2}) &= \cos 3t - \cos 3tu(t - \frac{\pi}{2}) \\ x(t) = \cos 3t\left(1 - u(t - \frac{\pi}{2})\right) &= \begin{cases} \cos 3t & , \quad 0 \le t < \frac{\pi}{2} \; , \\ 0 & , \quad 2\pi \le t < \infty \end{cases} . \end{aligned}$$



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for i = 1:201
x24(i) = cos(3*(i-1)*pi/200);
end
for i = 102:201
x24(i) = 0;
end
t = linspace(0,1,31);
z = linspace(0,pi,201);
plot(z,x24,'-')
xlabel('t','Fontsize',14)
ylabel('x(t)','Fontsize',14)
title(' graph of x(t) = cos3t(1-u(t-π/2))')
legend('response x(t)',1)
```