Solutions, 316-XI

February 28, 2003

1 Problem 6.3.14

Find the differential operator that annihilates e^{5x}.
Solution:
In general, the operator $D - r$ annihilates the exponential e^{rx} so the desired operator is $(D - 5)$.

2 Problem 6.3.23

Use the annihilator method to determine the form of a particular solution to the DE

$$y'' - 5y' + 6y = \left(D^2 - 5D + 6\right) y = e^{3x} - x^2 \ .$$

Solution:
The characteristic equation is $r^2 - 5r + 6 = 0$ with roots $r = 2, 3$. Since 3 is a root, try

$$y_p(x) = Axe^{3x} + Bx^2 + Cx + D \ .$$

To discover this with the annihilator method:
$(D - 3)$ annihilates e^{3x}
D^3 annihilates x^2
So we have:

$$(D-3)\, D^3\, (D-3)\, (D-2)\, y = D^3\, (D-2) \left(e^{3x} - x^2\right) = 0$$

Then the annihilator of the general solution is $D^3\,(D-3)^2\,(D-2)$ and the general solution y_g contains terms that are annihilated by each of these operators. Then

$$y_g = C_1 e^{3x} + C_2 e^{2x} + Axe^{3x} + Bx^2 + Cx + D \ .$$

To find the particular solution set $C_1 = C_2 = 0$.

3 Problem 6.3.28

Use the annihilator method to determine the form of a particular solution to the DE

$$\left(D^2 - 6D + 10\right) = e^{3x} - x \ .$$

Solution:

$$D^2 - 6D + 10 = (D-3)^2 + 1$$

so that the characteristic equation, $(r-3)^2 + 1 = 0$ has roots $r = 3 \pm i$. Now to the right hand side: the homogeneous solution here is

$$y_h = C_1 e^{3x} \cos x + C_2 e^{3x} \sin x \ .$$

The operator $D - 3$ annihilates e^{3x} and the operator D^2 annihilates x. So we write

$$D^2(D-3)\left((D-3)^2 + 1\right) y = D^2(D-3)(e^{3x} - x) = 0$$