

313 – Midterm I

Name:

September 26, 2011

INSTRUCTIONS:
WORK ALL PROBLEMS!
YOU MAY ONLY USE YOUR OWN BRAIN AND ONE PAGE OF NOTES.
Instructor: E.A. Coutsias

Problem	grade
1	
2	
3	
4	
5	
6	
7	
Total	

1. (20.)

(a) (10pts) Find all complex numbers z for which

$$\left| e^{-iz^2} \right| = 1$$

holds.

(b) (10pts) Find all values of z for which

$$|z + 4i| < 1 \ .$$

2. (20pts.) Find all values of z such that

(a) $e^{iz} = 1 + i\sqrt{3}$.

(b) $\cos z = 2i$.

(c) $z^i = -1$.

(d) $\log\left(i + \sqrt{z^2 + 3}\right) = -\frac{\pi}{2}i$.

3. (15pts.) Show that $u(x, y) = \cos x \cosh y$ is harmonic in some domain (which?) and find a harmonic conjugate $v(x, y)$.

4. (10pts.) Show that

$$\lim_{z \rightarrow \infty} \frac{z+1}{\sqrt{z^3-1}} = 0$$

by using the equivalence

$$\lim_{z \rightarrow \infty} f(z) = 0 \iff \lim_{z \rightarrow 0} f\left(\frac{1}{z}\right) = 0 .$$

5. (10pts.) Show that the function $f(z) = e^{\bar{z}}$ is nowhere differentiable.

6. (10pts) Find all values of

$$\sqrt{3 + 4i} + \sqrt{3 - 4i} .$$

7. (15pts) Use the Cauchy-Riemann equations to verify that the following function is entire:

$$f(z) = (z^2 - 2)e^{-ix}e^y .$$